ELASTIK BO‘LAK-BO‘LAKLI BIR JINSLI IKKI QATLAMLI PLASTINKANING TEBRANISHI

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Annotatsiya: Ushbu maqolada “Ikki qatlamli noelastik plastinkaning ko‘ndalang tebranishi umumiy tenglamasini tahlil qilish” nomli [3] maqolaga asoslanib, bo‘lak-bo‘lakli bir jinsli yelastik plastinkaning ko‘ndalang tebranishlari uchun umumiy tenglama hisoblab chiqilgan.

Kalit so‘zlar: tahlil; taqribiy; tebranish; ikki qatlamli, elastik plastinka; boshlang‘ich shartlar; siljish; tebranish tenglamasi.

Abstract: In this article, based on the article “Analysis of the general equation for transverse vibrations of a two-layer inelastic plate” [3], the general equation for transverse vibrations of a piecewise homogeneous elastic plate is calculated.

Keywords: analysis; approximation; vibration; two-layer, elastic plate; initial conditions; displacement; equation of vibration.

Аннотация: На основе статьи [3] под названием «Анализ общего уравнения поперечных колебаний двухслойной неупругой пластины» рассчитано общее уравнение поперечных колебаний кусочно-однородной упругой пластины.

Ключевые слова: анализ; примерно; вибрация; двухслойная, эластичная пластина; начальные условия; перемещение; колебательное уравнение.

Kirish

Bu masalani echish uchun bo‘lak-bo‘lakli bir jinsli plastinkaning tebranishning [3] maqolada berilgan (1) taqribiy tenglamasidan foydalanamiz va Q_j koeffitsientlar (2) formulalar bilan aniqlanadi hamda M_0 va M_1 viskoelastik operatorlar mos ravishda μ_0 va μ_1 Lyame koeffitsientlariga almashtiriladi.

$$Q_1\left(\frac{\partial^4 W}{\partial t^4}\right)+Q_2\left(\Delta\frac{\partial^2 W}{\partial t^2}\right)+Q_3\left(\Delta^2 W\right)+Q_4\left(\frac{\partial^6 W}{\partial t^6}\right)+Q_5\left(\Delta\frac{\partial^4 W}{\partial t^4}\right)+$$
$$+Q_6\left(\Delta^2\frac{\partial^2 W}{\partial t^2}\right)+Q_7\left(\Delta^3 W\right)=F_1(x,y,t). \qquad (1)$$

Plastinkalar yuzalarida tashqi yuklar yo‘q.

Masalaning qo‘yilishi va echish usullari.

Ikki qatlamli plastinka $0\le x\le l_1,\ \ 0\le y\le l_2$ maydonni egallagan bo‘lsa va uning barcha to‘rt chetlari sharnirli tayanchlar bo‘lsa, bu masalada W siljish uchun chegaraviy shartlar quyidagi ko‘rinishga ega:

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$$W = \frac{\partial^2 W}{\partial x^2} = \frac{\partial^4 W}{\partial x^4} = 0; \quad (x = 0; \quad x = l_1) \tag{2}$$

$$W = \frac{\partial^2 W}{\partial y^2} = \frac{\partial^4 W}{\partial y^4} = 0; \quad (y = 0; \quad y = l_2)$$

Plastinkaning majburiy tebranishlari nolga teng bo‘lmagan boshlang‘ich shartlar bilan qo‘zg‘atiladi, ya’ni [1]

$$W = \varphi(x, y); \quad \frac{\partial W}{\partial y} = \psi(x, y); \tag{3}$$

$$\frac{\partial^2 W}{\partial y^2} = \ldots = \frac{\partial^7 W}{\partial y^7} = 0.$$

φ va ψ funktsiyalar (3) ko‘rinishidagi shartlarni qanoatlantirishi kerak, ya’ni

$$\varphi = 0; \quad \psi = 0; \quad (x = 0; x = l_1; y = 0; y = l_2).$$

(1) tenglamaning echimini

$$W = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} W_{n,m}(t) \sin(\gamma_n x) \sin(\nu_m y), \tag{4}$$

shaklda izlaymiz.

(4) ifodani (1) tenglamaga qo‘yamiz. Yuqoridagilarni hisobga olib, $W_{n,m}(t)$ uchun oltinchi tartibli oddiy differensial tenglamani xosil qilamiz

$$\frac{d^6 W_{n,m}}{dt^6} + A_1 \frac{d^4 W_{n,m}}{dt^4} + A_2 \frac{d^2 W_{n,m}}{dt^2} + A_3 W_{n,m} = 0, \tag{5}$$

bu yerda A_j koeffitsiyentlar quyidagilarga teng bo‘ladi:

$$A_1 = \frac{h_0^2 Q_1 - \gamma Q_5}{h^2 Q_4}; \quad A_2 = -\gamma \frac{h_0^2 Q_2 - \gamma Q_6}{h^2 Q_4}; \tag{6}$$

$$A_3 = \gamma^2 \frac{h_0^2 Q_3 - \gamma Q_7}{h^2 Q_4}; \quad \gamma = \left(\frac{\pi h_0}{l_1} n\right)^2 + \left(\frac{\pi h_0}{l_2} m\right)^2.$$

(5) tenglamaning echimini quyidagi shaklda qidiramiz [2]

$$W_{n,m}(t) = W_0 \exp\left(\frac{b_0}{h_0} \xi \cdot t\right), \tag{7}$$

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bu yerda ξ ikki qatlamli elastik plastinkaning hususiy tebranish chastotalarini aniqlaydigan o‘lchovsiz chastota.

(7) ifodani (5) tenglamaga almashtirib, ξ chastota uchun oltinchi tartibli algebraik tenglamani olamiz.

$$\xi^6 + A_{10}\xi^4 + A_{20}\xi^4 + A_{30} = 0, \tag{8}$$

bu yerda

$$A_{10} = \frac{Q_1' - \gamma Q_5'}{Q_4'}; \quad A_2 = \gamma \frac{hQ_2' - \gamma Q_6'}{Q_4'}; \quad A_3 = \gamma^2 \frac{Q_3' - \gamma Q_7'}{hQ_4'}, \tag{9}$$

va

$$\begin{aligned}
Q_1' &= P_2^2(1+h\rho)^2;\\
Q_2' &= -2P_2^2(2(P_2D_0 + hD_1)(1+h\rho) + (P_2-1)((1+h) - (D_0 + hD_1\rho)));\\
Q_3' &= 4(P_2-1)(P_2D_0 + h^2D_1 + 2hP_2D_0);\\
Q_4' &= -\frac{1}{6}P_2^2((3h^2\rho^2 + (1+4h\rho))(2-D_0) +\\
&\quad + h^2P_2(3 + h\rho(h\rho+4))(2-D_1));\\
Q_5' &= -\frac{1}{6}P_2((2P_2(4D_0(1-D_0)+1) + (P_2-1)(4-D_0^2)) -\\
&\quad - P_2h^4\rho^2(2(4D_1^2 - 4D_1 - 1) - (P_2-1)D_1(2-D_1)) +\\
&\quad + 6h^2(\rho(4(P_2^2D_0 + D_1) + (P_2-1)(2P_2(1-D_0) - P_2D_1(2-D_0) +\\
&\quad + D_1(1+D_0))) + P_2(1+\rho^2)) +\\
&\quad + 2h(2P_2\rho(2 + 4D_0 - D_0^2) - h^2(2P_2 - P_2D_1 + 5D_1 - D_1^2)) + (P_2-1)(4-3D_0) +\\
&\quad + 2D_1(4-D_0)) + 2P_2h\rho^2D_0(4-D_1));\\
Q_6' &= \frac{1}{3}P_2(2D_0(3P_2 - 4D_0 - 1) + (P_2-1)(2 + 9D_0 - 3D_0^2)) +\\
&\quad + h_1^4P_2\rho(4D_1(1-2D_1) - 4D_1 + (P_2-1)D_1(3-D_1)) +\\
&\quad + 3h^2(4P_2D_0(P_2(1-D_1) - D_1) - (P_2-1)(2(P_2-1)D_1(1-D_0) -\\
&\quad - P_2(2 - D_0 - 4D_0D_1)) + P_2\rho(4D_1(1 + D_0 + P_2D_0) - (P_2-1)(6D_0D_1(P_2-1) -\\
&\quad - 6P_2D_0 + D_1))) + 2hP_2(2(2D_1(1+2D_0) + (P_2-1)(1 + 2D_0 - D_0^2)) +\\
&\quad + h_1^2(2(P_2-1) + D_1(P_2+3)) -\\
&\quad - 4P_2\rho D_0(1 + h_0^2(2(P_2-1)(1-D_1) + P_2D_1 + (1+D_1))));
\end{aligned} \tag{10}$$

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$$Q_7 = \frac{2}{3}(P_2 D_0(4D_0 - 5(P_2 - 1) + h_1^4 D_1(4D_1 - (P_2 - 1)) -$$
$$- 3h^2(8P_2 D_0 D_1 + (P_2 - 1)(3P_2 D_0 - (2P_2 + 1)D_0 D_1 - D_1(1 - D_1))) -$$
$$- 4hP_2 D_0(2D_1 + (P_2 - 1)) + h_1^2(2(P_2 - 1) + (P_2 + 1)D_1)));$$

(10)

hamda o‘lchamsiz parametrlar kiritiladi:

$$h = \frac{h_1}{h_0}; \quad \rho = \frac{\rho_1}{\rho_0}; \quad P_2 = \frac{\mu_1}{\mu_0}; b = \frac{b_1}{b_0}; \quad D_0 = \frac{1}{2(1-\nu_0)}; \quad D_1 = \frac{1}{2(1-\nu_1)};$$

$\mathrm{Im}\,\xi_{10}, \mathrm{Im}\,\xi_{20}, \mathrm{Im}\,\xi_{30}$ lar 8(2.1.20) tenglama ildizlarining mahum qiymatlari, bunda ildizlari

$$\xi_{1,2} = \pm \mathrm{Im}\,\xi_{10}; \quad \xi_{3,4} = \pm \mathrm{Im}\,\xi_{30}; \quad \xi_{5,6} = \pm \mathrm{Im}\,\xi_{40}.$$

(8) ning xarakteristik tenglama ildizlarining tabiatiga asoslanib, masalaning umumiy echimini quyidagishaklda yozishimiz mumkin

$$W_{n,m} = C_1 \cos\left(\frac{b_0}{h_0}\xi_1 \cdot t\right) + C_2 \sin\left(\frac{b_0}{h_0}\xi_1 \cdot t\right) + C_3 \cos\left(\frac{b_0}{h_0}\xi_2 \cdot t\right) +$$
$$+ C_4 \sin\left(\frac{b_0}{h_0}\xi_2 \cdot t\right) + + C_5 \cos\left(\frac{b_0}{h_0}\xi_3 \cdot t\right) + C_6 \sin\left(\frac{b_0}{h_0}\xi_3 \cdot t\right). \qquad (11)$$

C_j o‘zgarmaslarni aniqlash uchun biz boshlang‘ich shartlarning $\varphi(x,y)$ va $\psi(x,y)$ funktsiyalarini

$$\delta_{1(1)}^{(n,m)} = -\frac{b_0}{h_0}\mathrm{Re}\,\xi_{10}; \quad \delta_{1(2)}^{(n,m)} = -\frac{b_0}{h_0}\mathrm{Re}'\,\xi_{10}; \quad \delta_j^{(n,m)} = -\frac{b_0}{h_0}\mathrm{Re}\,\xi_{j0}; \quad (j = 2,3,4)$$
$$\alpha_{n,m} = \frac{b_0}{h_0}\mathrm{Im}\,\xi_{20}; \quad \beta_{n,m} = \frac{b_0}{h_0}\mathrm{Im}\,\xi_{30}; \quad \gamma_{n,m} = \frac{b_0}{h_0}\mathrm{Im}\,\xi_{40} \qquad (12)$$

shaklida taqdim etamiz.

(11) - (12) ifodalarni va (3) boshlang‘ich shartlarni (4) echimnga qo‘yib, C_j uchun quyidagi ifodalarni olamiz:

$$C_1 = \frac{\xi_2^2 \xi_3^2}{\xi_1(\xi_1^2 - \xi_2^2)(\xi_1^2 - \xi_3^2)} \frac{h_0}{b_0} \psi(x,y); \qquad (13)$$

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$$C_2 = -\frac{\xi_2^2\xi_3^2}{(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)}\varphi(x,y);$$

$$C_3 = -\frac{\xi_1^2\xi_2^2}{\xi_2(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)}\frac{h_0}{b_0}\psi(x,y);$$

$$C_4 = \frac{\xi_1^2\xi_3^2}{(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)}\varphi(x,y);$$

$$C_5 = \frac{\xi_1^2\xi_2^2}{\xi_3(\xi_2^2-\xi_3^2)(\xi_1^2-\xi_3^2)}\frac{h_0}{b_0}\psi(x,y);$$

$$C_6 = -\frac{\xi_1^2\xi_2^2}{(\xi_2^2-\xi_3^2)(\xi_1^2-\xi_3^2)}\varphi(x,y). \qquad (13)$$

(12) ni (11) - (4) iboralarga qo‘yib, chetlari sharnirli mahkamlangan bo‘lak-bo‘lakli bir hil jinisli ikki qatlamli elastik plastinkalarning majburiy tebranishlari masalasiga echim topamiz.

$$W = \sum_{n,m=1}^{\infty}\Big(\frac{\xi_2^2\xi_3^2}{(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)}\Big(\frac{h_0}{b_0\,\xi_1}\psi_{n,m}\sin\Big(\frac{b_0}{h_0}\xi_1 t\Big)+\varphi_{n,m}\cos\Big(\frac{b_0}{h_0}\xi_1 t\Big)\Big)-$$
$$-\Big(\frac{\xi_1^2\xi_3^2}{(\xi_1^2-\xi_2^2)(\xi_2^2-\xi_3^2)}\Big(\frac{h_0}{b_0\,\xi_2}\psi_{n,m}\sin\Big(\frac{b_0}{h_0}\xi_2 t\Big)+\varphi_{n,m}\cos\Big(\frac{b_0}{h_0}\xi_2 t\Big)\Big)+$$
$$+-\Big(\frac{\xi_1^2\xi_2^2}{(\xi_1^2-\xi_3^2)(\xi_2^2-\xi_3^2)}\Big(\frac{h_0}{b_0\,\xi_3}\psi_{n,m}\sin\Big(\frac{b_0}{h_0}\xi_2 t\Big)+\varphi_{n,m}\cos\Big(\frac{b_0}{h_0}\xi_3 t\Big)\Big)\otimes$$
$$\otimes\sin\left(\frac{\pi n}{l_1}x\right)\sin\left(\frac{\pi m}{l_2}y\right). \qquad (14)$$

Xususan, agar

$$\varphi(x,y)=\varphi_0\sin\left(\frac{\pi n}{l_1}x\right)\sin\left(\frac{\pi m}{l_2}y\right);\quad \psi=0. \qquad (15)$$

bo‘lsa, W uchun biz quyidagi ifodani olamiz

$$W(x,y,t)=\frac{\xi_2^2\xi_3^2}{(\xi_1^2-\xi_2^2)(\xi_1^2-\xi_3^2)}\cos\left(\frac{b_0}{h_0}\xi_1 t\right)\varphi_0\sin\left(\frac{\pi n}{l_1}x\right)\sin\left(\frac{\pi m}{l_2}y\right). \qquad (16)$$

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XULOSA

To‘rtburchak shaklidagi chetlari sharnirli tayanch bo‘lgan ikki qatlamli bo‘lak-bo‘lakli bir jinsli plastinkaning tebranishlari masalasida, ikkala qatlamlar elastik bo‘lganda, hisoblab chiqilgan chastota tenglamasi murakkab ildizlarga ega bo‘lib, uning haqiqiy qismi manfiy bo‘lib, plastinka tebranishining so‘nishini va mavhum qism esa tebranishning hususiy chastotalarini aniqlaydi.

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