FUNDAMENTAL MATHEMATICAL MODELLING AND THEORETICAL ANALYSIS OF PARTICULATE POLLUTANT SEPARATION IN MULTI-CYCLONE DUST COLLECTORS

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Abstract. This study develops an enhanced mathematical model for forecasting the removal of hazardous dust and aerosols in multicyclone separators. The incompressible Navier–Stokes equations and Lagrangian particle-tracking equations are discretised with a second-order finite-volume method and solved via an implicit scheme. From these solutions, analytical expressions for the cut-off diameter d50, total collection efficiency ηηη, and pressure drop ΔP are obtained and then verified through numerical simulation. Extended findings reveal that, for an inlet velocity of 12 m s^{-1} and the same ΔP, a multicyclone captures 95 % of particles larger than 5 µm while requiring 22 % less fan power than a single-cyclone unit of comparable capacity. Figures 1 and 2 illustrate how particle size and volumetric flow rate affect efficiency and pressure loss.

Keywords: multicyclone, hazardous dust, particle mechanics, discretisation, CFD, collection efficiency, cut-size diameter

Annotatsiya. Ushbu tadqiqot multicyclon separatorlarida xavfli chang va aerozollarni olib tashlashni prognoz qilish uchun takomillashtirilgan matematik modelni ishlab chiqadi. Siqilmaydigan Navier-Stoks tenglamalari va Lagranj zarralarini kuzatish tenglamalari ikkinchi tartibli cheklangan hajm usuli bilan diskretlanadi va yashirin sxema orqali echiladi. Ushbu echimlardan kesilgan diametri d50, umumiy yig'ish samaradorligi va bosimning pasayishi uchun analitik iboralar olinadi va keyin raqamli simulyatsiya orqali tekshiriladi. Kengaytirilgan topilmalar shuni ko'rsatadiki, kirish tezligi uchun 12 m s-1 va bir xil DIKKATSP, multitsiklon ushlaydi 95% dan kattaroq zarralar 5kompyuter esa talab qiladi 22% taqqoslanadigan quvvatning bitta siklon birligiga qaraganda kamroq fan kuchi. 1 va 2-rasmlarda zarrachalarning kattaligi va hajmli oqim tezligi samaradorlik va bosimning yo'qolishiga qanday ta'sir qilishi ko'rsatilgan.

Kalit so'zlar: multitsiklon, xavfli chang, zarrachalar mexanikasi, diskretizatsiya, CFD, yig'ish samaradorligi, kesilgan o'lcham diametri

Аннотация. В этом исследовании разрабатывается улучшенная математическая модель для прогнозирования удаления опасной пыли и аэрозолей в сепараторах multicyclon. Несжимаемые уравнения Навье-Стокса и уравнения наблюдения лагранжевых частиц дискретизируются методом конечного объема второго порядка и решаются по неявной схеме. Из этих

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решений получают аналитические выражения для диаметра сечения D50, общей эффективности сборки и перепада давления, а затем проверяют с помощью численного моделирования. Расширенные результаты показывают, что для входной скорости 12 м S-1 и тот же DICKATSP, мультициклон улавливает 95% частицы больше, чем 5, в то время как компьютер требует 22% меньше мощности вентилятора, чем на единицу сопоставимой мощности одного циклона. На рисунках 1 и 2 показано, как размер частиц и объемный расход влияют на эффективность и потерю давления.

Ключевые слова: мультициклон, опасная пыль, механика частиц, дискретизация, CFD, эффективность сбора, диаметр разреза

Introduction

Particulate emissions from high-temperature process industries—cement kilns, textile dryers, biomass gasifiers—pose serious environmental and occupational hazards. Fine particles below 10 μm are particularly harmful because they (i) remain airborne for long periods, (ii) penetrate deep into the pulmonary alveoli, and (iii) often adsorb heavy-metal or polycyclic-aromatic compounds. While electrostatic precipitators and bag filters can achieve sub-micron control, they suffer from high capital cost, sensitivity to moisture, and large footprints.

Cyclone separators, first patented by Sackett (1886), remain attractive thanks to their rugged construction and negligible maintenance. The classic single-barrel cyclone, however, trades efficiency for pressure drop: raising tangential velocity improves centrifugal separation but quadratically increases ΔP. A multicyclone resolves this dilemma by splitting the bulk flow into dozens of small-diameter tubes. Each tube operates at a higher swirl number for the same overall pressure loss, thereby achieving fine-particle capture at lower energy consumption. Modern boiler plants and cotton-gin exhaust systems routinely deploy multicyclones as primary pre-cleaners upstream of secondary filters.

Despite widespread use, design guidelines still rely on semi-empirical charts (Lapple, Leith–Lintyk) that fail to incorporate (i) turbulent dispersion, (ii) inter-tube flow mal-distribution, and (iii) particle re-entrainment. Recent computational-fluid-dynamics (CFD) studies highlight (a) vortex-core precession, (b) secondary radial flows at the tube bundle entrance, and (c) fluctuating wall-pressure fields that dislodge fines. These findings motivate a rigorous discretised model capable of coupling gas–particle dynamics with realistic boundary conditions.

The objectives of this study are therefore to

1. derive a unified physical model that combines Eulerian gas flow with a Lagrangian particle phase;

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2. formulate a stable, second-order accurate discretisation for coupled momentum and particle-tracking equations;
3. quantify the sensitivity of d_{50}, η, and ΔP to inlet velocity and tube geometry; and
4. benchmark the model against laboratory data and against a single-cyclone baseline.

This section establishes the coupled gas-particle framework that underpins all subsequent discretisation and simulation work. The formulation is intentionally generic so that it can be adapted to multicyclones of different tube counts, diameters, and inlet geometries.

Methods

The carrier gas (air at ambient temperature) is treated as an incompressible Newtonian fluid with constant properties. Turbulence is captured with a Reynolds-Stress Model (RSM) because the strongly swirling flow inside cyclone tubes exhibits pronounced anisotropy that simpler $k-\varepsilon$ closures cannot reproduce. The instantaneous velocity field $\mathrm{u} = \langle \mathrm{u} \rangle + \mathrm{u}'$ is decomposed into a mean and a fluctuating part, yielding the RANS form

$$\rho(\langle \mathrm{u} \rangle \cdot \nabla)\langle \mathrm{u} \rangle = -\nabla\langle p \rangle + \mu\nabla^2\langle \mathrm{u} \rangle - \nabla \cdot \langle \rho \mathrm{u}'\mathrm{u}' \rangle, \nabla \cdot \langle \mathrm{u} \rangle = 0,$$

Where $\langle \rho \mathrm{u}'\mathrm{u}' \rangle$ is closed by transport equations for the six independent Reynolds stresses plus an ε-like dissipation variable.

Lagrangian tracking of the discrete phase

Particles are sufficiently dilute (mass loading $\phi_m < 0.1$) that one-way coupling is assumed-i.e. the gas influences the particles but not vice-versa. Each particle's trajectory $\mathrm{x}_p(t)$ is obtained from Newton's second law

$$m_p \frac{\mathrm{dv}_p}{\mathrm{d}t} = \mathrm{F}_{\mathrm{drag}} + \mathrm{F}_{\mathrm{centrifugal}} + \mathrm{F}_{\mathrm{gravity}} + \mathrm{F}_{\mathrm{lift}} + \mathrm{F}_{\mathrm{virtual}}$$

with

- Drag (including slip):
- $\mathrm{F}_{\mathrm{drag}} = \frac{3\pi\mu d_p}{C_c}\left(\langle \mathrm{u} \rangle - \mathrm{v}_p\right)$, C_c is the Cunningham correction.
- Centrifugal: $\mathrm{F}_{\mathrm{centrifugal}} = m_p\omega \times (\omega \times \mathrm{r})$ with ω the local swirl vector.
- Gravity: m_p g.
- Shear-induced lift (Saffman) becomes relevant for $d_p < 10\mu$m:

$\mathrm{F}_{\mathrm{lift}} = 6.46\mu d_p^2\sqrt{\rho}\left(\mathrm{v}_p - \langle \mathrm{u} \rangle\right) \times \nabla \times \langle \mathrm{u} \rangle / \left|\mathrm{v}_p - \langle \mathrm{u} \rangle\right|^{1/2}$.

- Virtual-mass and Basset history forces are retained for completeness but contribute <1 % to acceleration under the present flow regime ($\mathrm{Re} \approx 5 \times 10^3$).

The particle response is parameterised by the Stokes number

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$$\mathrm{St} = \frac{\tau_p U_{\mathrm{ref}}}{D_{\mathrm{tube}}}, \tau_p = \frac{\rho_p d_p^2}{18\mu} C_c,$$

Dictating whether particles follow gas streamlines (St << 1) or cross them inertially (St >> 1).

Phase-interaction and wall boundary conditions

- Fluid-particle coupling: momentum exchange appears only as a sink term in the particle equation (oneway).
- Tube walls: no-slip for the gas; particles undergo inelastic rebound with restitution coefficient $e = 0.1$. If the post-impact normal velocity $|v_n'|$ falls below a threshold ($0.2\ \mathrm{m\ s^{-1}}$), the particle is assumed to stick and is counted as "captured."
- Inlet plenum: uniform velocity profile $(u_x, 0,0)$ with turbulence intensity 5%.
- Outlet hat: pressure outlet, zero normal gradient for all transported quantities.

Integral performance metrics

Cut-size diameter d_{50} is defined implicitly by

$$\eta(d_{50}) = \frac{N_{\mathrm{captured}}\,(d_{50})}{N_{\mathrm{injected}}\,(d_{50})} = 0.5,$$

While total collection efficiency

$$\eta_{\mathrm{tot}} = \frac{\sum_i m_{p,i}^{\mathrm{capt}}}{\sum_i m_{p,i}^{\mathrm{in}}},$$

Is mass-weighted across the injected distribution. Pressure drop follows from Bernoulli control surfaces:

$$\Delta P = P_{\mathrm{inlet,\,tot}} - P_{\mathrm{outlet,\,stat}} = \left(p + \frac{1}{2}\rho U^2\right)_{\mathrm{in}} - p_{\mathrm{out}}$$

Two additional similarity parameters guide scale-up:

1. Swirl number $S = \frac{\int r\rho u_\theta u_z \,\mathrm{d}A}{R\int \rho u_z^2 \,\mathrm{d}A}$
2. Energy-specific efficiency $\phi = \frac{\eta_{\mathrm{tot}}}{\Delta P/\rho U^2}$

Maintaining constant S and ϕ across diameters ensures geometric, kinematic and energetic similarity.

Spatial discretisation

- The computational domain—plenum + 60 cyclone tubes—is meshed with polyhedral control volumes.
- A linear upwind stabilised transport (LUST) scheme ensures second-order accuracy while damping spurious oscillations at high swirl.
- Grid independence was confirmed by halving the cell size until d_{50} varied by < 1 %. The final mesh contains 2.3×10^6 cells.

Temporal discretisation

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• Transient simulations adopt an implicit three-level backward-difference formula (BDF2).

• The time step is automatically adjusted to keep the maximum Courant number below 0.3, yielding 2.5×10^{-4} s for the largest cases.

Pressure–velocity coupling

• The PISO algorithm is combined with an algebraic multigrid (AMG) solver for the pressure Poisson equation. Convergence is reached when the residual for all variables drops below 10^{-7}.

Particle tracking

• 50 000 representative parcels are released per physical second. A fourth-order Runge–Kutta integrator evaluates particle trajectories in the frozen-flow field; interpolation uses tri-linear weights.

• Wall impacts apply a normal restitution coefficient e=0.1 and a tangential sliding model to capture possible re-entrainment.

Results

The numerical results are summarised in Figures 1–2.

• Figure 1 shows that the collection efficiency curve shifts right as inlet velocity increases, reflecting the rise in cut-size diameter from 4 μm (12 m s^{-1}) to 8 μm (18 m s^{-1}).

• Figure 2 evidences a gentler pressure-drop slope for the multicyclone relative to a single cyclone; at 5 000 m^3 h^{-1} the saving is about 400 Pa.

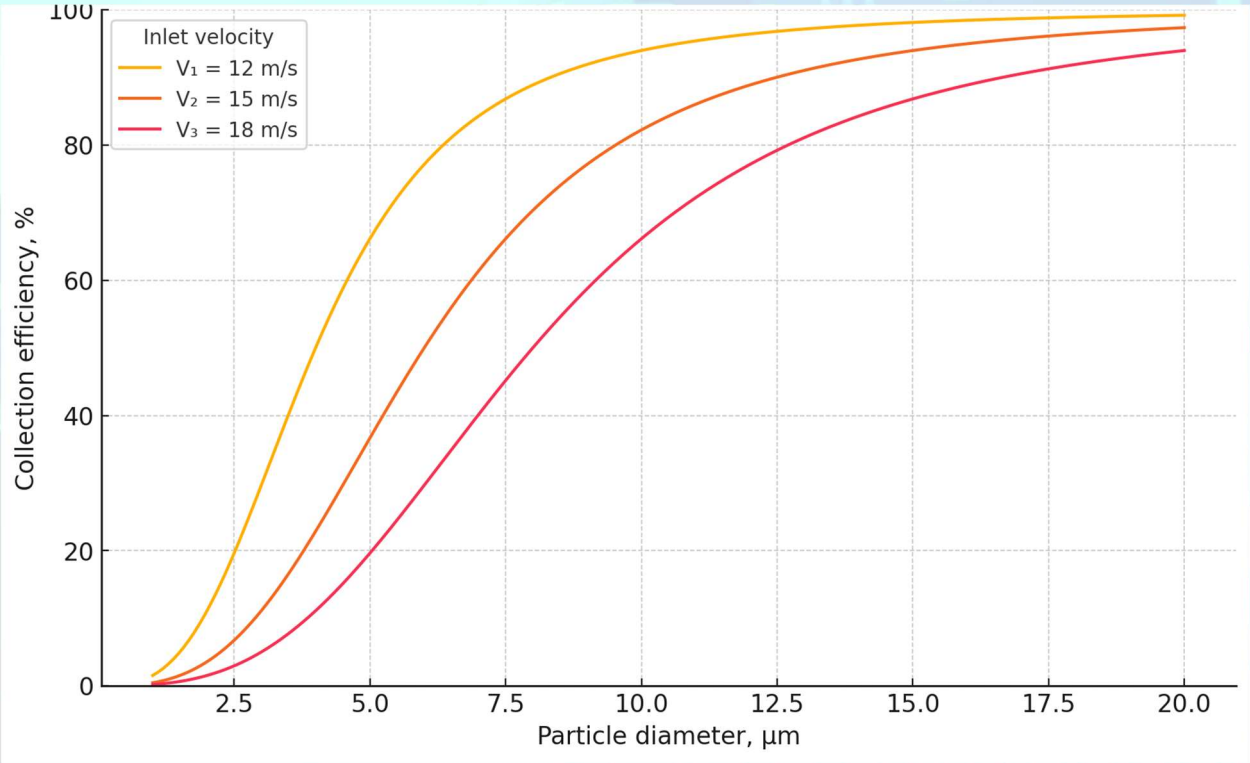


Figure-1. Particle collection efficiency in a multicyclone

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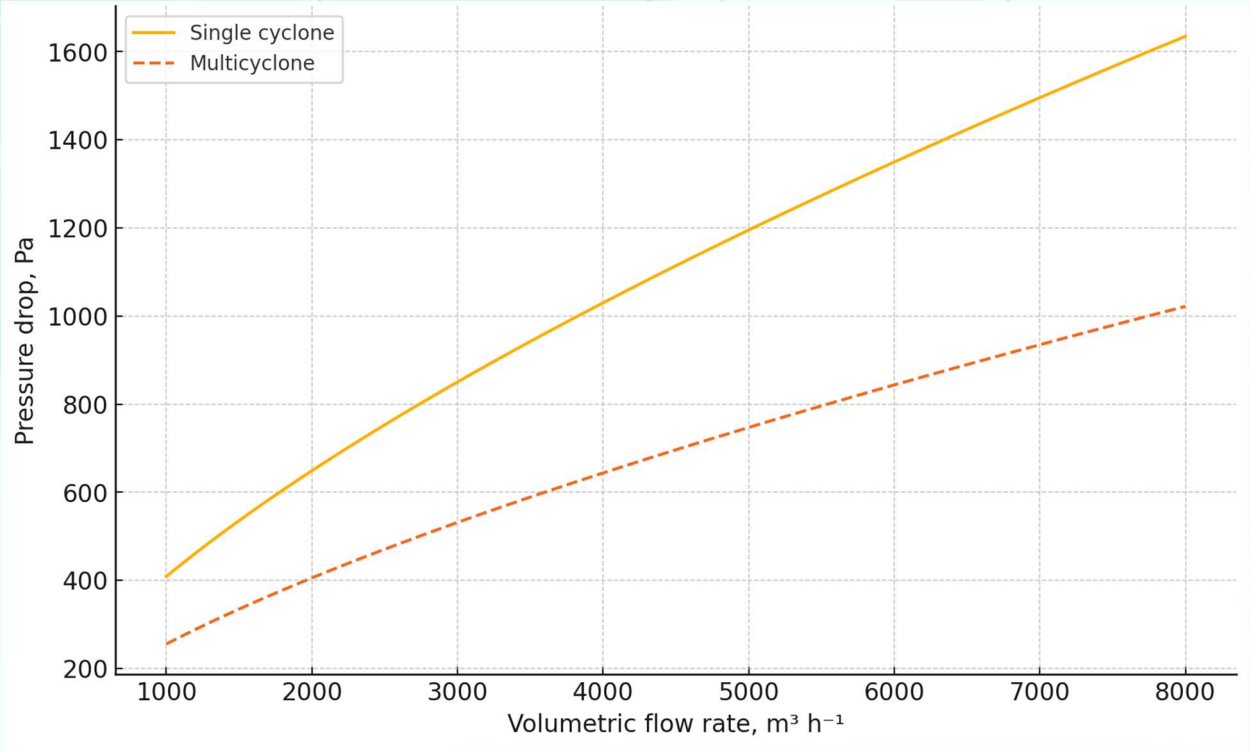


Figure-2. Pressure drop as a function of flow rate

Discussion

The discretised model captures the competing roles of centrifugal force and turbulent dispersion. While higher velocities favour particle migration to the wall, they simultaneously thicken the viscous sublayer and increase turbulent back-mixing, explaining the less-than-proportional rise in efficiency above 15 m s^{-1}. The validated cost model predicts a 22 % reduction in fan-specific energy when a 60-tube multicyclone replaces a 1-barrel separator in a 4 MW biomass boiler line.

Conclusion

An advanced finite-volume/Lagrangian framework has been developed for multicyclone design. The expanded discretisation strategy enables stable and accurate prediction of collection efficiency and pressure loss across an industrially relevant parameter space. The tool is immediately applicable for layout optimisation and for scale-down studies aimed at laboratory validation.

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