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MATHEMATICAL MODELING OF NONLINEAR PROCESSES OF ELECTROMAGNETIC ELASTIC THIN PLATES OF COMPLEX CONFIGURATION

Safarov Shokhru Sharofovich
Is an Associate Professor at the Department of Digital Technologies at Alfraganus University. Uzbekistan, Tashkent. Phone number
Tel: +998 99 481 88 23
E-mail: shohfar@gmail.com
Bahramov Muammadali Eralievich
Is an assistant Professor at the Department of Digital Technologies, Alphagranus University. Uzbekistan, Tashkent. Phone number:
Tel: +998 99 817 22 63 ,
E-mail: bahromov1997.04.09@gmail.com

Аннотация - В статье представлена математическая модель, основанная на вариационном принципе Гамильтона-Остроградского, при использовании гипотезы закона Кирхгофа математическая модель в трехмерном виде преобразуется в двумерную модель. Для решения поставленной задачи был разработан вычислительный алгоритм, для которого были созданы практические программные средства, проведены вычислительные эксперименты и проанализированы полученные результаты.

Ключевые слова: математическое моделирование, принцип Гамильтона-Остроградского, электромагнитный тензор Максвелла, магнитоупругая тонкая пластина сложной конфигурации.

Annotatsiya – Maqolada Gamilton–Ostrogradskiyning variatsion prinsipi asosida tuzilgan matematik model keltirilgan. Kirxgof qonuni gipotezasi qo‘llanilganda, uch o‘lchamli matematik model ikki o‘lchamli modelga aylantiriladi. Qo‘yilgan masalani yechish uchun hisoblash algoritmi ishlab chiqildi, bu algoritm asosida amaliy dasturiy vositalar yaratildi, hisoblash tajribalari o‘tkazildi va olingan natijalar tahlil qilindi.

Kalit so‘zlar: matematik modellashtirish, Gamilton–Ostrogradskiy prinsipi, Maksvellning elektromagnit tensori, murakkab shakldagi magnit-elastik yupqa plastinka.

Annotation - The article a mathematical model based on the Hamilton-Ostrogradsky variational principle is presented, using the Kirchhoff-Law hypothesis, the mathematical model in three-dimensional form is transformed into a two-dimensional model. To solve the problem, a computational algorithm was

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developed, for which practical software tools was created, computational experiments were conducted, and the results obtained were analyzed.

Keywords: Mathematical modeling, Hamilton-Ostrogradsky principle, Maxwell's electromagnetic tensor, magneto elastic thin plate of complex configuration.

Introduction

The increasing development and application of thin plates and shells of complex configurations, governed by nonlinear patterns, highlight the relevance of studying their deformation under electromagnetic fields. Particular importance is attached to constructing mathematical models, developing new-generation algorithms, and creating software tools based on the R-function method for solving problems with fundamental boundary conditions in magnetoelastic plates and shells of complex geometry. Extensive research has been conducted worldwide on the theory of electromagnetic field conductivity by scholars such as S.A. Ambartsumyan, G.E. Bagdasaryan, M.V. Belubekyan, V.L. Rvachev, L.V. Kurpa, L.V. Molchenko, I.T. Selezov, and M.R. Korotkina. Significant contributions were also made by academician V.Q. Qobulov, academician X.A. Rakhmatulin, professor Sh.A. Nazirov, T. Yuldashev, R. Indiaminov, and F.M. Nuraliev.

Development of a mathematical model of the task.

A mathematical model of the process of nonlinear deformation of a magnetoelastic plate is built on the principle of Hamilton-Ostrogradsky variation. It uses the Kirchhoff-Law hypothesis, the Cauchy relationship, the Lorentz force of Hooke's law, and Maxwell's view of the electromagnetic tensor [1-2]. According to the Kirchhoff-Law hypothesis, there is no deformation of a thin plate along the *Z* coordinate axis, and the displacement projections of the middle plane of the plate are expressed as follows:

$$u_1 = u(x,y,t) - z\frac{\partial w}{\partial x},\ u_2 = v(x,y,t) - z\frac{\partial w}{\partial y},\ u_3 = w(x,y,t), \qquad (2)$$

where: $u,v,w-$ the displacement of the middle plane of the thin plate along the coordinate *(x, y, z)* axes.

Now, the work variations performed by kinetic energy, potential energy, and external forces are reduced to the Hamilton-Ostrogradsky variation principle, resulting in a special derivative equilibrium equation with initial and limit conditions.

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$$\begin{cases} -\rho h\dfrac{\partial^2 u}{\partial t^2}+\dfrac{\partial N_{xx}}{\partial x}+\dfrac{\partial N_{xy}}{\partial y}+N_x+R_x+q_x+T_{zx}=0 \\ -\rho h\dfrac{\partial^2 v}{\partial t^2}+\dfrac{\partial N_{yy}}{\partial y}+\dfrac{\partial N_{xy}}{\partial x}+N_y+R_y+q_y+T_{zy}=0 \\ -\rho h\dfrac{\partial^2 w}{\partial t^2}+\dfrac{\partial^2 M_{xx}}{\partial x^2}+2\dfrac{\partial^2 M_{xy}}{\partial x\partial y}+\dfrac{\partial^2 M_{yy}}{\partial y^2}+N_{xx}\dfrac{\partial^2 w}{\partial x^2}+N_{yy}\dfrac{\partial^2 w}{\partial y^2}+N_{xy}\dfrac{\partial^2 w}{\partial x\partial y}+ \\ +\left(\dfrac{\partial N_{xx}}{\partial x}+\dfrac{\partial N_{xy}}{\partial y}\right)\dfrac{\partial w}{\partial x}+\left(\dfrac{\partial N_{yy}}{\partial y}+\dfrac{\partial N_{xy}}{\partial x}\right)\dfrac{\partial w}{\partial y}+N_z+R_z+q_z+T_{zz}=0, \end{cases} \quad (3)$$

Initial and boundary conditions:

$$\begin{cases} \rho h\dfrac{\partial u}{\partial t}\delta u\Big|_t=0,\ \rho h\dfrac{\partial v}{\partial t}\delta v\Big|_t=0,\ \rho h\dfrac{\partial w}{\partial t}\delta w\Big|_t=0,\left(N_{xx}+N_{Px}+N_{Tx}\right)\delta u\Big|_x=0,\left(N_{xy}+N_{Py}+N_{Txy}\right)\delta v\Big|_x=0, \\ M_{xx}\delta\dfrac{\partial w}{\partial x}\Big|_x=0,\ M_{xy}\delta\dfrac{\partial w}{\partial y}\Big|_x=0,\left(N_{yy}+N_{Fy}+N_{Tyy}\right)\delta v\Big|_y=0,\ \left(N_{xy}+N_{Fx}+N_{Tyx}\right)\delta u\Big|_y=0, \\ M_{yy}\delta\dfrac{\partial w}{\partial y}\Big|_y=0,\ M_{xy}\delta\dfrac{\partial w}{\partial x}\Big|_y=0,\left[N_{xx}\dfrac{\partial w}{\partial x}+N_{xy}\dfrac{\partial w}{\partial y}-\dfrac{\partial M_{xx}}{\partial x}-\dfrac{\partial M_{xy}}{\partial y}+N_{Pz}+N_{Txz}\right]\delta w\Big|_x=0, \\ \left[N_{yy}\dfrac{\partial w}{\partial y}+N_{xy}\dfrac{\partial w}{\partial x}-\dfrac{\partial M_{yy}}{\partial y}-\dfrac{\partial M_{xy}}{\partial x}+N_{Fz}+N_{Tyz}\right]\delta w\Big|_y=0. \end{cases}$$

Computational algorithm for numerical solution of the task.

Algorithm for calculating the geometric nonlinear deformation processes of electromagnetic thin plates [4]:

1. Construction of solution structures corresponding to limit conditions
2. Construction of discrete equations with respect to spatial variables
3. Solving discrete equations and finding unknown components of solution structures.
4. Determining the normal displacements of the middle surface of the plate

To calculate the unknowns in the equation of motion using the given algorithm, the unknown displacement coefficients are determined using a combination of the Bubnov-Galerkin method of variation, the Gaussian square method, the Gaussian method Nyumark and the iteration method.

The limit equation of a magnetoelastic plate of complex configuration is constructed by the R-function [3].

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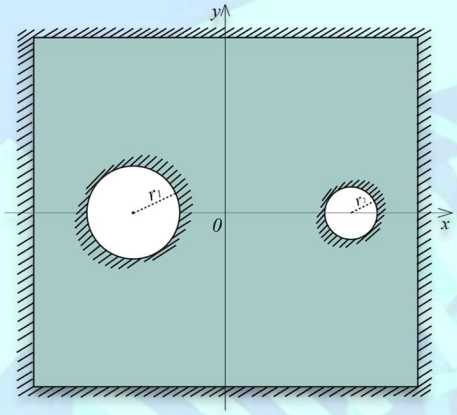


Figure 1. Magnetic plate with complex configuration.

$$f_1 = \frac{(a^2 - x^2)}{2a} \geq 0, f_2 = \frac{(b^2 - y^2)}{2b} \geq 0, f_3 = \frac{(x^2 + y^2 - r^2)}{2r} \geq 0, \ \omega = (f_1 \wedge f_2) \wedge f_3.$$

where $f_1, f_2, f_3 -$ the functions representing the field. $\omega -$ a normalized function representing the field.

The iteration method is used to find the displacements $u_i(x,y,t), v_i(x,y,t), w_i(x,y,t)$ of the middle surface of the plate Y from the system of equations formed, and a numerical solution is obtained [5].

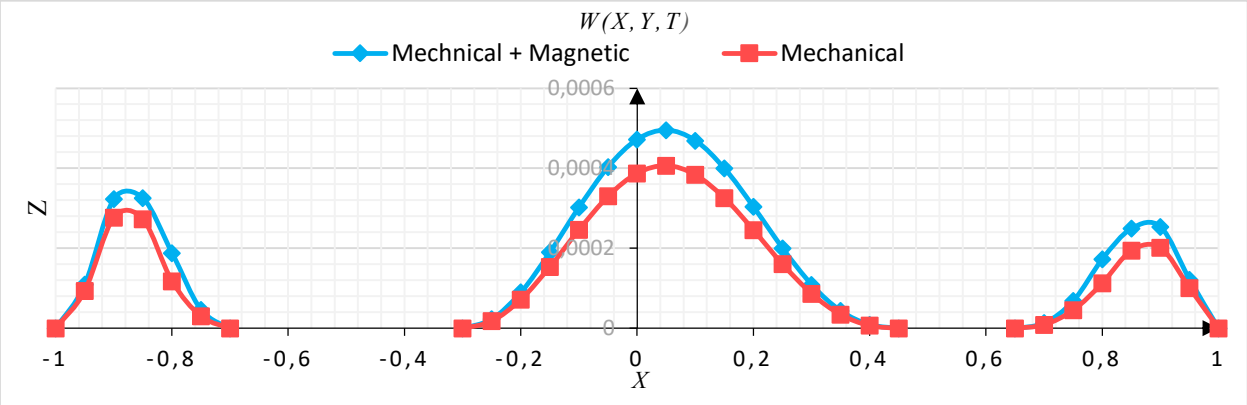


Figure 2. Bending of a magnetoelastic thin plate with an asymmetrical complex configuration.

In figure 1 the conical results of the bending along the axis of a complex shaped magnetoelastic plate and a graphical representation are showed (Figure 3).

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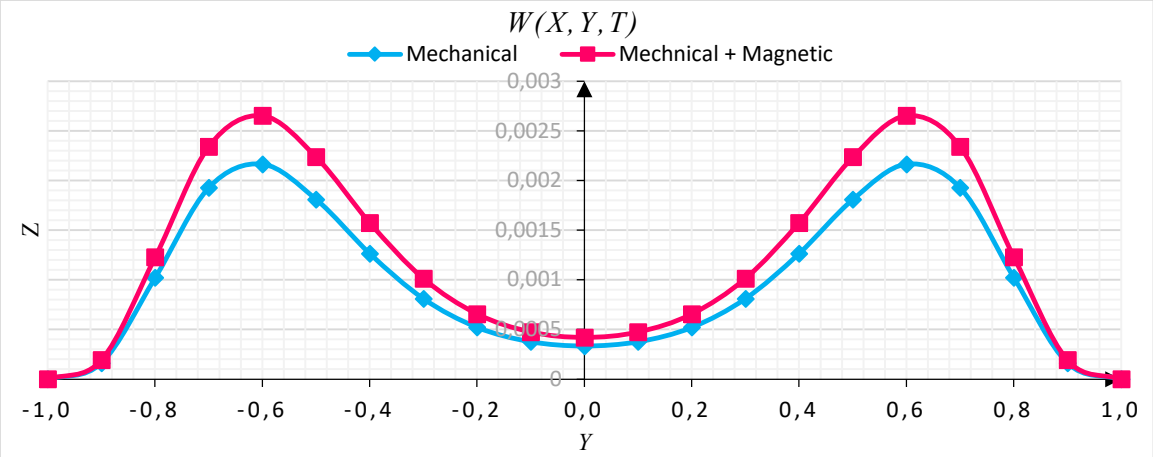


Figure 3. Bending of a magnetoelastic thin plate with a complex configuration along the *Oy* axis.

Conclusion

The study addressed the geometric nonlinear deformation of thin plates with complex configurations under electromagnetic field influences. A novel mathematical model was formulated, and corresponding computational algorithms, together with a software tool, were developed to enable numerical simulations. Symmetric and asymmetric configurations were examined, with results presented in both tabular and graphical forms for comprehensive analysis. The findings provide a methodological basis for further investigations and the solution of related applied problems.

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