# HYBRID MODEL OF MICROCLIMATE CONTROL USING PARTIAL DIFFERENTIAL EQUATIONS AND OPTIMIZATION METHODS

**Varlamova Lyudmila Petrovna**
*Doctor of Technical Sciences, professor National University of Uzbekistan,*
*E-mail: dimirel@gmail.com*
**Rahimova Mohira Muzaffar qizi**
*Master's student in the field of "Information Systems and Technologies" at the National University of Uzbekistan*
*E-mail: rahimovamohira730@gmail.com*

**Annotatsiya.** Maqolada ta’lim muassasalarida mikroiqlimni boshqarish uchun gibrid yondashuv taklif etilgan bo‘lib, unda xususiy hosilali tenglamalar (XHT) asosidagi modellashtirish, optimallashtirish usullari va IoT texnologiyalariga asoslangan prediktiv boshqaruv birlashtirilgan [1]. PDE havo oqimi, harorat va $CO_2$ dinamikasini yuqori aniqlikda tasvirlaydi, IoT esa real vaqt monitoringi va moslashuvchanlikni ta’minlaydi [1]. Metodologiya issiqlik balansi tenglamalari va PMV indeksini XHT-cheklangan optimallashtirish va MBB doirasiga integratsiya qiladi [1]. ESP32, DHT22 va MH-Z19 sensorlariga asoslangan pilot IoT tizimi barqaror qulaylikni ta’minlab, an’anaviy termostatga nisbatan 15% energiya tejash imkonini berdi [1]. Natijalar XHT modellarini IoT bilan integratsiya qilish HVAC samaradorligini, boshqaruv aniqligini va ichki muhit sifatini oshirishini tasdiqlaydi [1].

**Kalit so‘zlar:** ta’lim muassasalari, HVAC tizimlari, xususiy hosilali tenglamalar (XHT), optimallashtirish, modelga asoslangan bashoratli boshqaruv (MBB), energiya tejamkorligi, IoT.

**Аннотация.** В статье представлен гибридный подход к управлению микроклиматом в учебных помещениях, объединяющий моделирование на основе уравнений в частных производных (PDE), оптимизационные методы и IoT-технологии предиктивного управления [1]. PDE обеспечивают высокую точность описания потоков воздуха, температуры и динамики $CO_2$, в то время как IoT обеспечивает мониторинг в реальном времени и адаптивность [1]. Методология интегрирует уравнения теплового баланса и индекс PMV в задачу PDE-оптимизации с использованием MPC [1]. Пилотная IoT-система на базе ESP32, DHT22 и MH-Z19 обеспечила стабильный комфорт и до 15% экономии энергии по сравнению с термостатным управлением [1]. Результаты подтверждают, что интеграция моделей PDE с IoT повышает эффективность HVAC, точность регулирования и качество внутренней среды [1].



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**Ключевые слова:** образовательные помещения, системы HVAC, уравнения в частных производных (УЧП), оптимизация, модельно-предиктивное управление (МПУ), энергосбережение, IoT.

**Abstract.** This article presents a hybrid approach to microclimate control in educational spaces by combining partial differential equations (PDE)-based modeling**,** optimization methods**,** and IoT-enabled predictive control [1]. PDEs describe airflow, temperature, and $CO_2$ dynamics with high accuracy, while IoT provides real-time monitoring and adaptability [1]. The methodology integrates thermal balance equations and the PMV index into a PDE-constrained optimization and MPC framework [1]. A pilot IoT system with ESP32, DHT22, and MH-Z19 sensors achieved stable comfort and up to 15% energy savings compared to thermostat-based regulation [1]. The results confirm that integrating PDE models with IoT enhances HVAC efficiency, control precision, and indoor comfort [1].

**Keywords:** educational spaces, HVAC systems, partial differential equations (PDE), optimization, model predictive control (MPC), energy saving, IoT.

## Introduction

Modern educational buildings require not only efficient energy consumption but also a high-quality indoor environment, which directly affects the health and academic performance of students [2]. Traditional HVAC systems are usually controlled centrally or by static rules and often ignore changes in room configuration, human activity, and local spatial characteristics [3–4].

In recent years, there has been growing interest in hybrid models of microclimate control that combine physical partial differential equations (PDE) with optimization methods [3]. This approach makes it possible to account for the spatial distribution of temperature, air velocity, and $CO_2$ concentration, which is especially important for classrooms with high occupant density [5].

The use of PDE models in HVAC systems provides the opportunity for precise description of airflow and heat transfer dynamics, while integration with optimal control methods (e.g., MPC or adjoint-based techniques) increases regulation efficiency [3–5]. Furthermore, the development of digital twin technologies and IoT introduces new possibilities for adaptive control, personalization of microclimate parameters, and energy savings [7–9].

However, applying PDE-based approaches is associated with high computational complexity and the need for numerical solution methods, which limits their real-time applicability [7]. Therefore, research into hybrid models that combine PDE, optimization methods, and intelligent algorithms represents a promising direction in the development of effective microclimate control systems for educational buildings.



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The indoor microclimate is described by equations that account for heat transfer, air convection, and pollutant concentration. In three-dimensional CFD models, the conservation equations of mass, momentum, and energy (the Navier–Stokes equations) are solved, allowing spatial distributions of air velocity and temperature to be obtained [7]. The main equations include the heat transfer and convection equations with sources of heat and $CO_2$, as well as the Navier–Stokes equations (linearized or turbulent models if required). In particular, $CO_2$ concentration is often modeled using the convection–diffusion equation with a source term representing human emissions: for example, [4] describe $CO_2$ as the solution of a convective PDE, where the source ($CO_2$ exhaled by people) is defined by the output of an aperiodic ODE.

Unlike distributed PDE models, zonal ODE models ("resistance–capacitance") represent a room as one or several well-mixed zones. Such models are simple and computationally fast (solved by ordinary ODEs), but they do not capture the spatial heterogeneity of temperature fields or local airflow patterns [3-4]. Weekly and colleagues note that CFD models provide the most accurate representation of airflow but require precise room geometry and significant computational resources [3]. Zonal models, on the other hand, only account for mass and energy balances over the volume in a simplified manner, without reflecting, for example, local air jets from ducts or uneven vertical temperature distribution [1]. Thus, distributed PDEs capture microclimate details (temperature gradients, local flows, etc.), whereas ODE models provide a coarser, averaged picture.

Modern microclimate models also take human dynamics into account. For example, $CO_2$ models may incorporate class schedules: it is well known that indoor $CO_2$ concentration is proportional to the number of occupants and ventilation efficiency. The literature discusses algorithms for estimating occupancy from $CO_2$ signals via coupled PDE–ODE models [2-10]. Moreover, the microclimate includes other factors (lighting, humidity, human activity), but in the context of climate control, temperature and $CO_2$ are often considered the main indicators of indoor air quality [2].

**Methodology**

**1. Mathematical Model**

The classroom is represented as a spatio-temporal domain **Ω**, within which the following fields are defined:

- $T(x,t)$ — temperature,
- $C(x,t)$ — $CO_2$ concentration,
- $u(x,t)$ — airflow.

**The following equations are used:**



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- **Convection–diffusion (for temperature and $CO_2$):**

$$\frac{\partial \phi}{\partial t} + u\nabla\phi = D_\phi \Delta\phi + S_\phi(x,t)$$

**•CFD modeling of airflow (simplified Navier–Stokes equations):** provides a detailed spatial distribution of the flow [3].

**•Optimization Problem Formulation**

**Objective function:**

$$J = \int_0^{T_f} (\alpha E(u,q) + \beta \int_\Omega \left[ (T - T_{opt})^2 + (C - C_{opt})^2 \right] dx) dt$$

where:

- $E(u,q)$ — energy consumption,
- $T_{opt}, C_{opt}$ — target comfort parameters,
- $\alpha, \beta$ — weighting coefficients.

**Constraints:** PDE system, boundary conditions, and control limitations. The solution is obtained via **PDE-constrained optimization** using **adjoint methods** [1,9].

**Model Predictive Control (MPC)**

MPC optimizes control over a prediction horizon, adapting to changing conditions. The effectiveness of this strategy in the design of HVAC control systems has been confirmed by numerous studies [9].

**PDE-based approaches to HVAC control can be schematically divided into several stages:**

**•Mathematical model construction.**

The governing equations are selected to describe the dynamics of temperature, airflow, and $CO_2$. Typically, heat transfer and convection equations are used for air (e.g., the Navier–Stokes equations under the assumption of laminar flow in large volumes), and the convection–diffusion equation is applied for $CO_2$ [6–7]. Heat and $CO_2$ sources (e.g., occupants, equipment) are introduced as additional terms in the equations. The model accounts for the geometry of the room and ventilation systems (for example, the wall sandwich structure, location of ventilation shafts, etc.)

**•PDE discretization.**

A sliding **space–time scheme** is applied. For the numerical solution of the equations, a computational grid is constructed (finite element or finite volume mesh). A balance between accuracy and computational complexity is required — the literature describes mesh refinement techniques in critical zones, where the


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number of cells may reach millions [5, 7]. Solving PDEs can be computationally expensive; [4] emphasize that high resolution is essential for accurately reproducing airflow and $CO_2$ concentration.

- **Formulation of the optimization problem.**

A quality functional is introduced, for example, the total energy consumption of the HVAC system, penalties for deviations of temperature and $CO_2$ from desired values (comfort, regulatory standards), and constraints on system operating modes. The optimization problem with PDE dynamic constraints is formulated as **PDE-constrained optimization** [3]. The objective function may include compressor energy, the cost of cooling/heating, and constraints on the allowable parameter ranges.

- **Solution of the optimization problem.**

The adjoint equation method is typically used to evaluate the gradients of the functional with respect to the control variables. An adjoin system of equations is introduced (analogous to backpropagation in neural networks), and after solving it, derivatives of JJJ with respect to the control parameters are obtained. This gradient-based approach enables the use of fast optimization methods (e.g., gradient descent, quadratic programming, etc.) [3–4].

- **Integration with control (MPC).**

The resulting optimization framework can be embedded into a model predictive control (MPC) scheme. At each time step, the model predicts the future behavior of the microclimate, while the optimizer determines the optimal action (e.g., damper positions, fan speeds). An important advantage of the PDE model is the ability to control not only global variables (such as overall room temperature) but also local parameters such as airflow velocity or direction at specific ventilation outlets [2, 4]. MPC with a PDE-based model can, for instance, adjust the speed of individual fans or the position of dampers to locally correct the climate — something difficult to achieve with homogeneous ODE-based models [2].

- **Digital twin**

As part of the methodology, many studies propose the use of digital twins — dynamic virtual replicas of buildings in which the physical model (PDE) is synchronized with sensor data and the control system [9]. A digital twin can dynamically update the state of the indoor environment (temperature, $CO_2$, etc.) based on IoT sensor inputs and perform real-time optimization [5, 8]. Such a system makes it possible to predict microclimate changes under various scenarios (e.g., schedule shifts, window openings) and to preselect optimal HVAC settings in advance.

**Distributed PDE Approach and Future Perspectives**


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The distributed PDE-based modeling approach has both advantages and limitations.

**Advantages.** PDE models provide a detailed description of the spatial and temporal dynamics of indoor climate, explicitly accounting for room geometry and localized sources of heat and $CO_2$ [3-6]. This enables zone-specific climate control and allows rapid response to local disturbances such as drafts or the temporary clustering of occupants. Moreover, hybrid PDE frameworks are easily extendable: additional equations (e.g., for humidity or particulate matter) or parameters (e.g., material thermal inertia) can be incorporated without altering the core structure.

**Limitations.** The main drawback lies in the high computational cost. Solving nonlinear PDEs (e.g., Navier–Stokes equations) at full spatial resolution for real-scale rooms requires large meshes with millions of cells and significant computational resources, which hinders real-time operation [3-7]. Furthermore, PDE-based models involve a large number of physical parameters (thermal conductivity coefficients, convection velocities, etc.) that must be carefully calibrated. As noted in [1], the use of full nonlinear models increases the applicability of the controller but entails substantial computational expenses and necessitates a "careful selection of parameters and algorithms."

**Comparison with alternatives.** Zonal ODE models are much simpler, easier to identify, and are already used in a number of MPC systems. However, they neglect localized effects: for example, strong airflows near a fan or a heat source in a corner cannot be captured by a single energy balance equation [3-9]. While ODE models save computational time, they compromise predictive accuracy. Rule-based control (RBC), the simplest approach (e.g., switching on/off ventilation depending on temperature), is convenient but typically results in lower energy efficiency and poor adaptability to changing conditions. In contrast, optimization-based approaches such as hybrid MPC have been shown to outperform RBC in terms of both comfort and energy performance, albeit at the cost of more complex infrastructure [12].

**Future perspectives.** A promising direction is the integration of PDE models with IoT and digital technologies. Sensor networks enable real-time model updating: a building's digital twin can receive temperature, $CO_2$, humidity, and occupancy data and dynamically adapt its state [13,14]. Such digital twins are capable of "learning" user preferences and environmental changes, thus enabling adaptive HVAC operation that ensures both energy savings and improved comfort [13, 15–16]. Recent studies [15–16] demonstrated that coupling digital twins with deep learning models (e.g., LSTM with attention mechanisms) allows personalized thermal comfort prediction and further enhances control efficiency.


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Another key aspect is the inclusion of class schedules and occupant behavior in the models. For instance, occupancy density data (from schedules or $CO_2$ sensors) can be integrated into the model to proactively adjust ventilation parameters. Research on educational buildings emphasizes the necessity of a multidisciplinary approach that combines physical models, AI algorithms, and user data. In particular, [10] recommend developing modular data frameworks, leveraging AI-based optimization, and incorporating BIM and IoT technologies for dynamic indoor climate management.

In summary, the integration of PDE models with digital twins and machine learning holds strong potential: PDEs provide the physical foundation, while IoT and ML enable adaptability and personalization. At the same time, it is crucial to address uncertainties and modeling errors stemming from external condition variability and parameter inaccuracies. As noted in [11-16], incorporating posterior uncertainty quantification into PDE frameworks can significantly improve the reliability of HVAC control.

**Conclusion**

The conducted study has shown that hybrid models of microclimate control based on partial differential equations (PDE) and optimization methods hold significant potential for improving energy efficiency and enhancing indoor environmental quality in educational spaces. Unlike traditional ODE-based models and rule-based algorithms, the PDE approach makes it possible to account for spatial heterogeneity of temperature, humidity, and $CO_2$ concentration, as well as building geometry and occupant dynamics.

The integration of PDE modeling with optimization methods (such as MPC and adjoint-based approaches) opens opportunities for the development of intelligent HVAC systems capable of predicting microclimate changes and adaptively controlling ventilation, heating, and cooling. In this context, digital twins and IoT systems play an essential role by enabling real-time model updating and personalized comfort management.

Nevertheless, the practical implementation of PDE–optimization schemes faces several challenges: high computational complexity, the need for precise parameter calibration, and requirements for sensor network infrastructure. These challenges can be addressed through advances in numerical methods, the use of cloud computing, and the application of machine learning algorithms to accelerate computations.

Thus, the hybrid PDE–optimization model of microclimate control represents a promising direction that combines physical accuracy, adaptability, and intelligent algorithms. Its development will not only reduce energy consumption


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but also create a healthier and more comfortable educational environment, directly influencing student productivity and well-being.

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