ADVANCING METHODOLOGICAL AND MATHEMATICAL APPROACHES FOR MODELING THE DECISION-MAKING PROCESS OF THE DIGITAL BUSINESS ECOSYSTEMS

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Abstract: This article proposes a mathematical methodology for improving decision-making in business ecosystems by analyzing their structure, dynamics, and sensitivity to internal, micro-, and macro-environmental changes. A new algorithm for identifying linearly independent elements of digital ecosystems and interpreting their influence through eigenvector analysis is developed. The method determines how financial indicators affect the system by evaluating the contribution of eigenvectors to overall performance. Yandex’s 2023 data is used as a representative case to demonstrate the practical applicability of the approach. The findings provide guidance for top management in identifying critical impact points and making informed strategic decisions. Integrating this methodology with big data analytics and AI technologies enables a flexible and scalable tool for future managerial applications.

Keywords: business ecosystem; matrix models; linearly independent elements; vector analysis; ecosystem properties; financial indicators; decision-making.

Аннотация: В данной статье предлагается математическая методология для совершенствования процесса принятия решений в бизнес-экосистемах путем анализа их структуры, динамики и чувствительности к изменениям во внутренней, микро- и макросреде. Разработан новый алгоритм идентификации линейно независимых элементов цифровых экосистем и интерпретации их влияния посредством анализа собственных векторов. Метод определяет, как финансовые показатели воздействуют на систему, оценивая вклад собственных векторов в общую результативность. В качестве

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репрезентативного примера использованы данные «Яндекса» за 2023 год, что демонстрирует практическую применимость подхода. Полученные результаты предоставляют руководству высшего звена рекомендации по выявлению критических точек воздействия и принятию обоснованных управленческих решений. Интеграция данной методологии с аналитикой больших данных и технологиями искусственного интеллекта обеспечивает гибкий и масштабируемый инструмент для будущих управленческих приложений.

Ключевые слова: бизнес-экосистема; матричные модели; линейно независимые элементы; векторный анализ; свойства экосистемы; финансовые показатели; принятие решений.

Annotatsiya: Ushbu maqolada biznes ekotizimlarida qaror qabul qilish jarayonini takomillashtirish uchun ularning tuzilishi, dinamikasi hamda ichki, mikro va makromuhitdagi o‘zgarishlarga sezgirligini tahlil qilishga asoslangan matematik metodologiya taklif etiladi. Raqamli ekotizimlarning chiziqli jihatdan mustaqil elementlarini aniqlash hamda ularning ta’sirini eigenvektor (o‘z vektorlari) tahlili orqali talqin qilishga xizmat qiluvchi yangi algoritm ishlab chiqilgan. Taklif etilgan usul, eigenvektorlarning umumiy samaradorlikka qo‘shayotgan hissasini baholash orqali moliyaviy ko‘rsatkichlarning tizimga qanday ta’sir ko‘rsatishini aniqlash imkonini beradi. Metodologiyaning amaliy qo‘llab bo‘lishini namoyish etish uchun Yandex kompaniyasining 2023-yilga oid ma’lumotlari namunaviy holat sifatida o‘rganilgan. Olingan natijalar yuqori boshqaruv bo‘g‘ini uchun tanqidiy ta’sir nuqtalarini aniqlash va asosli boshqaruv qarorlarini qabul qilishda qo‘llanilishi mumkin bo‘lgan tavsiyalarni taqdim etadi. Ushbu metodologiyaning katta ma’lumotlar analitikasi va sun’iy intellekt texnologiyalari bilan integratsiyasi uni kelajakdagi boshqaruv amaliyotlari uchun moslashuvchan va kengaytiriladigan vositaga aylantiradi.

Kalit so‘zlar: biznes ekotizimi; matritsa modellari; chiziqli mustaqil elementlar; vektor tahlili; ekotizim xususiyatlari; moliyaviy ko‘rsatkichlar; qaror qabul qilish.

Introduction

This article explores how mathematical tools—specifically matrices of unlimited size — can be applied to model and enhance the decision-making processes within modern business ecosystems. In recent years, business ecosystems have attracted significant attention in both academic and practice-oriented fields due to their ability to unite diverse participants, technologies, and services around shared value creation. Today, many producers of technologically complex products adopt an ecosystem-based approach throughout the entire product lifecycle, from development to implementation and replication. Global examples such as Apple and Amazon illustrate how companies design interconnected platforms that align

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with consumer needs, offering cross-compatible products and services at competitive prices and supported by advanced digital sales technologies.

Large industrial companies also apply ecosystem principles. Siemens, for instance, integrates ecosystem capabilities in signaling technologies and software systems such as Signaling X and Desigo CC. In Russia, some of the most rapidly developing digital ecosystems — Yandex, Mail.ru Group and Sber — demonstrate the potential of ecosystem models at scale. Sber has formed one of the largest databases in the region through decades of user interactions, enabling the identification of behavioral trends essential for informed managerial decisions. These examples highlight that in today's era of rapid technological advancement, ecosystem-driven integration provides companies with a higher and more adaptive form of organizational development.

The relevance of such research stems from the rapid expansion of business ecosystems, driven primarily by reduced transaction costs and the strengthening of internal linkages within the customer base. As companies expand through ecosystem logic, they redirect customer needs toward their own services and products, achieving higher operational stability and synergy. The synergistic effect occurs because ecosystem participants jointly produce outcomes that none could achieve independently. This prompts researchers to examine how an organization can optimally enter an ecosystem. Identifying the ideal integration point within the organizational life cycle allows firms to expand strategic planning horizons, avoid stagnation or bankruptcy risks, and equip management with tools for flexible strategic decision-making.

The study also emphasizes the growing importance of synthetic, interdisciplinary fields and nature-like technologies that span across mathematics, economics, and management. When modeling decision-making based on the influence of financial indicators on ecosystem properties, several economic mechanisms analogous to biological ecosystem processes were examined. These analogies reveal that the optimal integration moment in the life cycle of an organization can be mathematically determined through indicators that reflect the ecosystemic characteristics of the entity. Such analysis provides a foundation for managerial decisions regarding integration into broader, higher-order business systems. This research contributes a new perspective in scientific literature by proposing that ecosystem readiness can be quantified.

Theoretical Background

The concept of business ecosystems, which forms the foundation of this research, reflects a complex network of participants whose interactions generate collective value exceeding the capabilities of individual organizations. The development of ecosystem theory in economics has deep historical roots. As early

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as the 1920s, A. Bogdanov in *Tectology: The Universal Organizational Science*, proposed examining organizational systems through principles of order, disorder, and mutual influence between internal and external components. Later in 1935, A. Tansley introduced the term "ecosystem" initially in biological sciences. Its adaptation to the economic field occurred much later when J. Moore in 1999, used biosimilar analogies to conceptualize evolving networks of firms that co-create and co-develop value. Moore's definition remains central today: a business ecosystem consists of organizations and individual actors that form mutually supportive relationships and undergo continuous evolution.

Recent literature has expanded the conceptualization, formation, and development mechanisms of business ecosystems. Studies have examined their foundational theory, structural development, and governance models. Researchers have also addressed issues of competition and cooperation, entrepreneurial behavior within ecosystems and the role of innovation in shaping ecosystem trajectories. Comprehensive reviews have explored entrepreneurial ecosystems both in mature and emerging markets, highlighting differences in institutional environments and growth potential. Sustainability, understood as a critical ecosystem property, has also received considerable attention, emphasizing resilience and long-term viability. Strategic management research has focused on how ecosystems create unique competitive advantages through shared capabilities, co-specialization and resource orchestration.

Despite these substantial contributions, several gaps remain. Existing classifications and terminology have been discussed extensively, yet the behavioral responses of ecosystems to managerial interventions are still insufficiently understood. Studies on organizational life cycles, crisis management, and bankruptcy prediction models offer relevant insights, but they do not fully address how organizations should determine the optimal moment to enter or integrate into a business ecosystem. This transition point is crucial for strategic planning, as it may prevent stagnation or decline and enhance long-term strategic adaptability.

Moreover, many decision-making tools currently applied in ecosystem management rely on expert judgment and average statistical indicators derived from limited samples. Such methods inherently carry subjectivity and often fail to provide robust, data-driven guidance for complex strategic decisions. In an era characterized by the widespread availability of big data and advanced computational methods, significant potential remains untapped for mathematical modeling approaches capable of capturing the multidimensional nature of ecosystem interactions.

The present research addresses these gaps by proposing a novel algorithm that uses mathematical tools—specifically automated computation of matrices of

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unlimited size—to evaluate the impact of financial indicators on the ecosystem's properties. This approach expands the dimensionality of analysis, enabling the inclusion of a broad set of indicators and property combinations. Such flexibility allows decision-makers to model complex interdependencies and examine how various organizational attributes contribute to ecosystem behavior. Furthermore, the ability to expand matrices indefinitely makes this method well-suited for integration with big data analytics and self-learning neural network models, which can identify hidden patterns and generate non-trivial solutions for managerial decision-making.

As a pilot study, this research represents one of the first attempts to quantitatively analyze how changes in financial indicators affect the properties of business ecosystems using mathematical modeling inspired by biological analogies. The methodology enables constructing influence matrices that incorporate unlimited numbers of decomposed properties and allows diverse scenario-based calculations. When the dataset grows significantly, the use of trained neural networks becomes necessary to process inputs, adjust parameter weights, and generate refined outputs.

From a practical standpoint, business ecosystems involve numerous dynamic internal and external relationships among participants, where roles and benefits may shift rapidly. The proposed methodology provides a tool for balancing diverse stakeholder interests while identifying optimal points for managerial interventions. Understanding the strength of ecosystem properties and their dependence on financial performance indicators will help organizations maintain system balance and pursue sustainable development.

To achieve these objectives, the research focuses on several key tasks:

- developing mathematical tools for managerial decision-making;
- selecting and validating indicators that represent core ecosystem properties;
- ranking the influence of these indicators;
- performing model calculations using the example of Yandex as a representative digital ecosystem;
- identifying impact vectors that shape overall system mechanisms.

The central hypothesis assumes that determining the optimal transition point for organizational integration into a business ecosystem significantly improves strategic planning effectiveness and enhances long-term system sustainability.

Materials and Methods

This research examines the largest Russian business ecosystems as its primary objects, including Sberbank, Yandex, T-Bank, and MTS. Based on a comparative review of their structure, maturity, and internal connectivity, Yandex was selected as the most illustrative case, as it demonstrates well-defined

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ecosystem properties and a developed set of interrelated components. The Yandex ecosystem continuously expands its service groups—ranging from blogging platforms and multimedia libraries to e-commerce marketplaces, taxi and car-sharing services, and cloud infrastructure. These interconnected services evolve similarly to biological cenoses, reinforcing one another through shared data flows, user interactions, and platform synergies.

The Runet—the Russian-speaking segment of the global Internet—acts as the biotope for this ecosystem. With its large and behaviorally diverse user base, the Runet provides a stable environment for Yandex to accumulate vast amounts of behavioral, transactional, and preference-based data. Modern communication technologies and large-scale data centers allow Yandex to collect and process user actions in real time, thereby optimizing service offerings and strengthening inter-component integration. Official financial and analytical reporting from Yandex N.V. for 2023 serves as the primary empirical source for evaluating ecosystem properties, such as integrity, sustainability, self-regulation, self-reproduction, and emergence.

All data used in the analysis were obtained from open public sources and processed through specialized matrix-calculation services. To ensure reliability, the methodology requires access to complete retrospective company reports that include all indicators necessary for characterizing ecosystem properties, particularly financial and operational indicators reflecting internal and external interactions.

The research methodology combines general scientific methods—such as factor analysis and expert evaluation—with specialized techniques involving analogy construction. Of particular importance is the second similarity theorem, which allows translating biological ecosystem properties into quantifiable economic indicators. This translation enables the mathematical modeling of ecosystem behavior.

In engineering and technical sciences, eigenvalues and eigenvectors are widely used to analyze complex systems by decomposing large processes into smaller, more manageable components. They support structural analysis, communication system design, and mechanical modeling. A. Kotelkin and M. Musin's monograph *Influence Matrices* provides the theoretical basis for applying such matrix methods to analyze interconnected systems. Applying eigenvector-based analysis to management decision-making within business ecosystems is novel and represents one of the first attempts to adapt these mathematical tools to organizational strategy.

Based on the theoretical characteristics of business ecosystems, this research identifies a set of performance indicators that determine the strength of each

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ecosystem property. The working hypothesis assumes that each property is shaped by a group of measurable characteristics, where positive dynamics indicate a strengthening of that property.

The indicators selected for the model include:

- Total revenue – reflecting the scale of ecosystem expansion;
- Profitability based on adjusted EBITDA – indicating cost efficiency and internal cohesion;
- Net profit – showing resilience to internal and external shocks;
- Share of the core company (Yandex Search) in the Russian market;
- Total number of users, capturing ecosystem reach and growth potential;
- Core revenue;
- Revenue from non-core services;
- Intra-segment revenue, which indicates internal resource redistribution and structural coordination.

From this set, the most relevant indicators were assigned to specific ecosystem properties:

- **Integrity** reflects the degree to which components collectively contribute to the ecosystem's objective function of maximizing revenue and minimizing costs. Since integrity depends on coordinated interactions and efficient resource use, profitability (EBITDA margin) serves as the most appropriate indicator.
- **Self-reproduction** refers to stable, long-term growth driven by the ecosystem's ability to generate new components or reinforce existing ones. Total revenue is the primary indicator, as it reflects the scale and momentum of overall expansion.
- **Sustainability** denotes the ecosystem's ability to maintain development despite disruptive influences. Net profit dynamics—measured as the ratio between the current and previous year—serve as a proxy for the ecosystem's capacity to withstand instability while following its strategic trajectory.
- **Self-regulation** involves internal redistribution of resources among components to maximize efficiency. Intra-segment revenue best captures these internal flows, demonstrating how resources move within the system to balance supply, demand, and operational efficiency.
- **Emergence** represents the appearance of new properties arising from interactions within the ecosystem. This research proposes evaluating emergence through the ratio of core revenue to service-based revenue, which reflects the ecosystem's ability to generate new value streams beyond its initial core functionality.

To determine the aggregated impact of these indicators on ecosystem properties, the research applies the law of self-preservation, which requires the

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ecosystem to maintain a stable combination of integrity, self-regulation, sustainability, self-reproduction, and emergence. This integrated property ensures long-term viability and creates a basis for further development.

The research also draws on Le Chatelier's principle of equilibrium, which states that systems counteract external changes by moving toward a new equilibrium. Applied to business ecosystems, this principle offers a theoretical framework for understanding how ecosystems evolve under external market pressures or internal structural shifts. Thus, the dynamic development model derived from this principle supports the mathematical modeling of ecosystem behavior adopted in this study.

To quantify the sustainability, the indicative indicators shown in Table 1 are used.

Table 1.

Defining indicative indicators for business ecosystem properties.

Property	Indicative Indicators
Integrity	Profitability (based on adjusted EBITDA)
Self-reproduction	Total revenue
Sustainability	Change in net profit of the system
Self-regulation	Intra-segment calculations
Emergence	Ratio of core revenue to service revenue

The self-preservation mechanism of a business ecosystem is based on maintaining a stable level of internal order and continuously counteracting destructive influences. These influences create disorganization (entropy), and the ecosystem's sustainability depends on whether its stabilizing resources exceed destabilizing pressures.

This balance can be expressed as

$$(C_1 + C_2) \geq (D_1 + D_2)$$

where *D1* represents internal destabilizing factors such as changes in behavior strategies, communication structures, management errors, shifts in the number of participants and transformations in core properties.

D2 reflects external destabilizing forces, including inflation, fluctuations in customer demand, changes in user preferences and loyalty, political or geopolitical instability, and economic crises.

On the opposite side, *C1* corresponds to internal stabilizing resources—effective managerial decisions, reconfiguration of internal links, development of competencies, and regulatory measures.

C2 includes external resources such as attracting investors, political support, and expanding the network of beneficiaries.

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The property of self-preservation reaches its highest expression at a dynamic equilibrium point that still allows development and supports the synergistic interaction of ecosystem properties. The synergy arising from the optimal realization of these properties increases the system's overall potential and determines opportunities for future growth. Importantly, a business ecosystem cannot remain in perfect equilibrium; it must constantly adapt and evolve, fully manifesting its capacities for self-reproduction, emergence, and long-term sustainability.

The general algorithm of the calculation part is shown in the next figure (Figure 1).

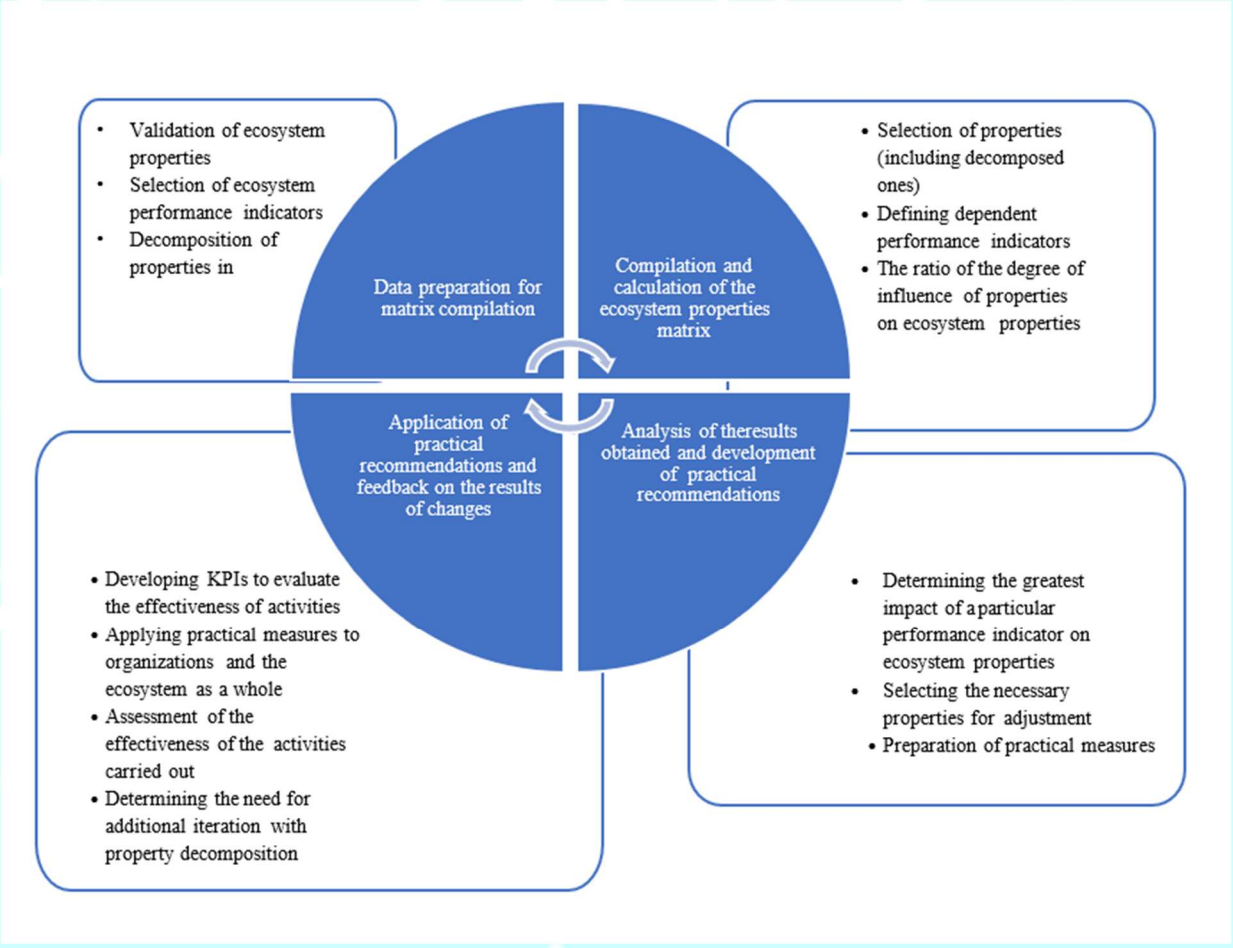


Figure 1. The general algorithm of the calculation

Results

To implement the proposed concept in practice, a methodology was developed to identify the point of maximum concentration of ecosystem properties, which reflects the most effective state for maintaining these properties and ensuring further development. Each key property of the business ecosystem—integrity, self-reproduction, sustainability, self-regulation, and emergence—is represented by one indicative indicator that best characterizes it. In the first iteration of the authors' model, every property is influenced directly by its indicator and indirectly, though more weakly, by the others. To measure this influence, a ranking method was

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applied to justify the significance of each indicator, with higher ranks reflecting lower influence.

Table 2.

Ranking the impact of indicators on the properties of the business ecosystem.

Ecosystem Properties	Integrity	Self-Reproduction	Sustainability	Self-Regulation	Emergence
Integrity	-	1	2	3	4
Self-reproduction	2	-	4	1	3
Sustainability	1	4	-	3	2
Self-regulation	3	2	1	-	4
Emergence	4	1	3	2	-

After ranking the indicators, the authors distributed their influence across all ecosystem properties. Based on the definitions, they justified how each property affects the others. Integrity, which depends on the contribution of every element to the ecosystem's overall outcome, is most strongly influenced by sustainability, followed by self-reproduction, sustainability, self-regulation, and emergence. A similar approach was applied to the remaining properties. To quantify these influences, a linear probability scale was initially used, assigning values from 1 to 0.2 by rank. Since a linear model is insufficient for real market conditions, the authors adapted it using the gamma distribution, widely applied in finance, insurance, reliability analysis, and biological modeling.

For the selected set of probabilities, the authors determined the required parameters of the gamma distribution, represented by the random variables α and β. Using the linear probability array (x = 0.2, 0.4, 0.6, 0.8, 1.0), the calculated parameters were α = 3.60 and β = 0.17. Based on these values, the authors computed both the distribution function and the probability density of the gamma distribution, with the corresponding results presented in Table 3.

Table 3.

Calculation of the gamma distribution

Rank	Linear Probability	Distribution Function	Probability Density
5	0.2	0.05464	0.74462
4	0.4	0.28388	1.39212
3	0.6	0.55548	1.23191
2	0.8	0.75950	0.80257
1	1	0.88172	0.44209

Next, applying the normal–gamma probability distribution, the authors calculated how each indicator affects the remaining four properties, thereby incorporating both the synergistic interaction among ecosystem properties and the

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probabilistic component of influence. The resulting distribution curve is presented in Figure 2.

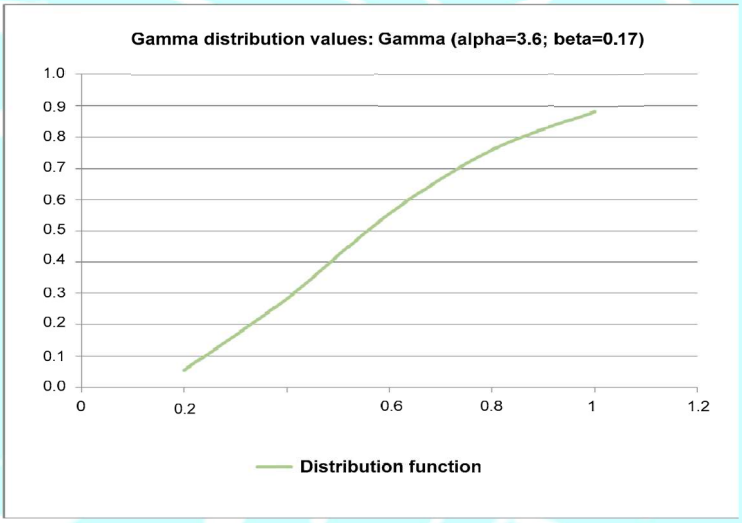


Figure 2. Gamma distribution indicators.

To convert the influence ranks into linear probabilities, the authors applied probability coefficients (S_n) to each indicator listed in Table 2. This made it possible to determine the numerical influence value for every indicator. Using the normal–gamma distribution law and the ranked influences shown in Table 2, the authors calculated the corresponding (S_n) values, which are presented in Table 3.

Next, based on the normal–gamma probability distribution, the authors quantified how each indicator affects the other four properties. This approach allowed them to incorporate both the synergistic interaction among ecosystem properties and the probabilistic nature of these influences.

The resulting matrix was constructed using indicators for a single selected year. To illustrate the methodology, the authors provided a matrix template in Table 4, containing one indicative indicator for each ecosystem property.

Table 4.

Template for the matrix of the impact of indicators on the properties of the business ecosystem

Indicator	Property of a Business Ecosystem				
	Integrity	Self-Reproduction	Sustainability	Self-Regulation	Emergence
Share S_{RO} Profitability (adjusted EBITDA), RO	X				

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Share S_R Total revenue, R		X			
Share $S_{\triangle NP}$ Change in the net profit of the system, ΔNP			X		
Share S_{IS} Intra-segment calculations, IS				X	
Share $S_{R_{core}/R_{serv}}$ The ratio of core revenue to service revenue R_{core}/R_{serv}					X

Based on the influence ranks from Table 2 and the probability distribution values from Table 3, the authors calculated the coefficients reflecting the impact of each indicator on the corresponding ecosystem property. The final results of these calculations are summarized in Table 5.

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Table 5.

Determining the coefficients of ecosystem indicators

Ranks	Integrity	Self-Reproduction	Sustainability	Self-Regulation	Emergence
Rank	1	2	3	4	5
S_{RO}	0.88172	0.75950	0.55548	0.28388	0.05464
Rank	3	1	5	2	4
S_R	0.55548	0.88172	0.55548	0.75950	0.28388
Rank	2	5	1	4	3
$S_{\triangle NP}$	0.75950	0.55548	0.88172	0.28388	0.55548
Rank	4	3	2	1	5
S_{IS}	0.28388	0.55548	0.75950	0.88172	0.55548
Rank	5	2	4	3	1
$S_{R_{core}/R_{serv}}$	0.55548	0.75950	0.28388	0.55548	0.88172

To identify the necessary managerial actions that guide a business ecosystem toward its strategic objectives, it is essential to establish an appropriate management model. Although a business ecosystem possesses self-regulating properties, this does not eliminate the need for targeted managerial influence. Instead, it allows corrective actions to restore equilibrium more quickly and with minimal resource expenditure. Therefore, the authors emphasize that management intervention should be minimized, yet not completely excluded, to ensure proper behavioral adjustment of the ecosystem. Based on this principle, the study proposes a practical management mechanism suitable for both decision makers and key stakeholders.

The management model is described as a controlled process represented by the equation

$$AX = \lambda X,$$

where (A) is the initial state of the ecosystem, (X) is the control impact, and (λX) represents the resulting effect of managerial influence. The system state can be expressed as

$$A = X\lambda X^{-1}$$

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where (λ) is the eigenvector reflecting the synergistic effect of management actions. Since ecosystem properties interact nonlinearly, management decisions generate a synergetic response.

The authors introduce the concept of "system energy," drawing an analogy with physical and biological systems, where internal energy reflects the combined influence of structural and dynamic components. Using ecosystem properties—integrity, self-reproduction, sustainability, and self-regulation—they apply the conservation and distribution of this "energy" within the system. In this framework, the sum of eigenvalues corresponds to the total system potential, while their distribution reflects each property's contribution. For sustainability analysis, the matrix of impacts was transposed and recalculated, with results summarized in Tables 6 and 7.

Table 6.

Matrix of the impact of indicators on the properties of the Yandex business ecosystem based on the results of work in 2023

Indicators	2023	Integrity	Self-Reproduction	Sustainability	Self-Regulation	Emergence
Rank		1	2	3	4	5
Probability		0.88172	0.7595	0.55548	0.28388	0.05464
Profitability (adjusted EBITDA), %	12.1	10.669	9.190	6.721	3.435	0.661
Rank		3	1	5	2	4
Probability		0.55548	0.88172	0.05464	0.7595	0.28388
Total revenue, mln. rub.	800,125	444,453.4	705,486.2	43,718.83	607,694.94	227,139.485
Rank		2	5	1	4	3
Probability		0.7595	0.05464	0.88172	0.28388	0.55548
Growth rate of the net profit of the system, %	46	34.937	2.513	40.559	13.058	25.552
Rank		4	3	2	1	5
Probability		0.28388	0.55548	0.7595	0.88172	0.05464
Intra-segment calculation	131,949	37,457.68	73,295.03	100,215.27	116,342.07	7209.69

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s, mln. rub.						
Rank		5	2	4	3	1
Probability		0.05464	0.7595	0.28388	0.55548	0.88172
The ratio of core revenue to service revenue, %	56	3.060	42.532	15.897	31.107	49.376

Table 7.

Transposed influence matrix (with rounded values)

Indicators	**Profitability (Adjusted**	**Total Revenue,**	**Change in the Net Profit of**	**Intra-Segment Calculations,**	**The Ratio of Core Revenue to Service**
	EBITDA), %	**mln. rub.**	**the System, %**	**mln. rub.**	**Revenue, %**
2023	12.1	800,125	46	131,949	56
Integrity	11	444,453	35	37,458	3
Self-reproduction	9	705,486	3	73,295	43
Sustainability	7	43,719	41	100,215	16
Self-regulation	3	607,695	13	116,342	31
Emergence	1	227,139	26	7210	49

Further, the matrix A_{mxn} presented in Table 7 in terms of finding eigenvalues and eigenvectors was calculated. The calculation was performed using an online service. The calculation results are presented below.

$$A_{m\times n} = \begin{pmatrix} 11 & 444453 & 35 & 37458 & 3 \\ 9 & 705486 & 3 & 73295 & 43 \\ 7 & 43719 & 41 & 100215 & 16 \\ 3 & 607695 & 13 & 116342 & 31 \\ 1 & 227 & 26 & 7210 & 49 \end{pmatrix}$$

The sum of the spectrum of the matrix is 821,929 and is calculated as the sum of the elements of the main diagonal:

$$\Lambda = a_{11} + a_{22} + a_{33} + a_{44} + a_{55}$$

where the following definition holds:

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A_{mn} is the value of the matrix cell of column m of row n. The eigenvectors of the matrix A are as follows:

$$v_1 \approx \begin{pmatrix} 6.384 \\ -0 \\ -3.116 \\ 0.001 \\ 1 \end{pmatrix}, \text{ the eigenvalue } \lambda_1 = -16.371$$

$$v_2 \approx \begin{pmatrix} -1.961 \\ -0 \\ -0.691 \\ +0 \\ 1 \end{pmatrix}, \text{ the eigenvalue } \lambda_2 = -29.602$$

$$v_3 \approx \begin{pmatrix} -0.100 \\ -0 \\ 0.552 \\ -0 \\ 1 \end{pmatrix}, \text{ the eigenvalue } \lambda_3 = 63.213$$

$$v_4 \approx \begin{pmatrix} -1.667 \\ -0.748 \\ 13.174 \\ 6.705 \\ 1 \end{pmatrix}, \text{ the eigenvalue } \lambda_4 = 48559.946$$

$$v_5 \approx \begin{pmatrix} 69.416 \\ 112.037 \\ 19.767 \\ 103.638 \\ 1 \end{pmatrix}, \text{ the eigenvalue } \lambda_5 = 773292.609$$

To determine how each indicator affects the full set of ecosystem properties, the authors used the eigenvectors derived from the matrix in Table 7. Calculating these eigenvectors makes it possible to identify the key indicators whose variations exert the strongest influence on ecosystem properties and contribute most to preserving the system's "energy." For clarity, all matrix-based calculations are presented in tabular form. The resulting eigenvector matrix ($A'_{m\times n}$) is summarized in Table 8 for further analysis.

$$A_{m\times n} = \begin{pmatrix} 6.384 & -1.961 & -0.1 & -1.667 & 69.416 \\ 0 & 0 & 0 & -0.748 & 112.037 \\ 3.116 & -0.691 & 0.552 & 13.174 & 19.767 \\ 0.001 & 607695 & 13 & 6.705 & 103.638 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

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Using the obtained data, the authors calculated the contribution of each eigenvector to the system mechanism A_v according to formula,

$$A_{v_m} = \frac{\lambda_m}{\Lambda}$$

Where λ_m represents the eigenvector value for indicator and Λ denotes the spectrum of matrix $A_{m\times n}$. The results in Table 8 show that "intra-segment calculations" and "the ratio of core revenue to service revenue" exert the strongest influence on the system, with the second indicator playing the dominant role.

Table 9 further summarizes the horizontal sums of eigenvector values, confirming that these two indicators have the greatest combined impact on all five ecosystem properties.

Table 8.

Eigenvectors of the impact of indicators on the properties of the business ecosystem and their contribution to the mechanism of the system

Indicators	Profitability (Adjusted EBITDA), %	Total Revenue, mln. rub.	Change in the Net Profit of the System, %	Intra-Segment Calculations, mln. rub.	The Ratio of Core Revenue to Service Revenue, %
Integrity	6.384	−1.961	−0.1	−1.667	69.416
Self-reproduction	0	0	0	−0.748	112.037
Sustainability	3.116	−0.691	0.552	13.174	19.767
Self-regulation	0.001	0	0	6.705	103.638
Emergence	1	1	1	1	1
Eigenvalues	−16.371	29.602	63.213	48,559.946	773,292.609
Contribution of eigenvalues to the system mechanism, %	−0.0020	0.0036	0.0077	5.9080	94.0827

Table 9.

The sum of the values of the eigenvectors of the impact of indicators on the properties of the business ecosystem

Indicators	Profitability (Adjusted EBITDA), %	Total Revenue, mln. rub.	Change in the Net Profit of the System, %	Intra-Segment Calculations, mln. rub.	The Ratio of Core Revenue to Service Revenue, %
Integrity	6.384	−1.961	−0.1	−1.667	69.416

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Self-reproduction	0	0	0	−0.748	112.037
Sustainability	3.116	−0.691	0.552	13.174	19.767
Self-regulation	0.001	0	0	6.705	103.638
Emergence	1	1	1	1	1
The sum of the values of the eigenvector	10.501	−1.652	1.452	18.464	305.858

To assess how each indicator affects the ecosystem's properties, the authors analyzed the structure of its eigenvectors. This was done by dividing each column value in Table 9 by the sum of its eigenvectors. Negative values were squared to eliminate sign effects. The resulting normalized table (Table 10) allowed verification that each column's share sums to 1. For clarity, cell values ranging from 0 to 0.72284 were color-coded. Focusing on columns 4 and 5—identified earlier as the most influential—the authors located the maximum share values, which revealed the dominant impact on specific properties. The results show that "intra-segment calculations" exert the strongest influence on the sustainability property, while the "ratio of core revenue to service revenue" most strongly affects self-reproduction. These quantified influence levels help identify key financial indicators for each ecosystem property. For example, modifying intra-segment connections can directly weaken or strengthen overall system stability by shifting the balance of forces that maintain equilibrium.

Table 10.

Structure of eigenvectors of influence of indicators on the properties of a business ecosystem

Indicators	Profitability (Adjusted EBITDA), %	Total Revenue, Million Rubles	Change in the Net Profit of the System, %	Intra-Segment Calculations, Million Rubles	The Ratio of Core Revenue to Service Revenue, %
Integrity	0.6079	0.72284	0.0076	0.0125	0.2270
Self-reproduction	0.0000	0.0000	0.0000	0.0025	0.3663
Sustainability	0.2967	0.089752	0.2318	0.7788	0.0646
Self-regulation	0.0001	0.0000	0.0000	0.2017	0.3388
Emergence	0.0952	0.187969	0.7606	0.0045	0.0033
The verification	1	1	1	1	1

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amount of the share					

The obtained results support the initial hypotheses and align with observed trends in how financial indicators influence the development or stagnation of a business ecosystem. To validate the model, the authors compared the sum of the matrix spectrum with the sum of the eigenvalues. As previously calculated, the spectrum total is 821,929. The sum of eigenvalues derived from Table 8 is identical— 821,929. This equality confirms the internal consistency and validity of the proposed model.

Discussion

The mathematical results obtained in this study have both theoretical and practical significance, as they reveal the internal mechanism of how business ecosystems function under the influence of various economic factors and managerial interventions. The hypothesis proposed in the research is supported by the existence of the gravity matrix and the results validated through retrospective data. The calculations performed according to the developed methodology demonstrate an effective approach to working with ecosystem indicators. These results can be applied by managers and ecosystem participants to strengthen or regulate specific properties over time, depending on internal and external environmental conditions, making the methodology a practical tool.

As noted earlier, calculations can be performed using different sets of decomposed properties. Users may link these properties to various indicators of ecosystem activity available from selected sources, which makes the methodology adaptive. However, careful attention must be paid to the correctness of indicator selection, since inaccurate alignment between indicators and ecosystem properties may lead to erroneous gravity calculations and ultimately distort strategic decision-making.

During the study, business ecosystems were selected based on the presence of key properties, components, and clearly defined internal and external linkages, including a distinct core. The method can also function as part of a machine-learning system using big data and an expanding diagnostic matrix, improving the accuracy of managerial recommendations. The mathematical basis facilitates future automation of the algorithm and helps identify favorable points in an organization's lifecycle for change. Nevertheless, large datasets must be verified, as unverified data may generate cumulative errors in neural-network calculations. Overall, the developed method provides a new tool for digitizing business ecosystems and supports strategic planning by identifying optimal points for lifecycle extension.

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