A WONDERFUL PROOF OF THEOREMS ON CERTAIN INEQUALITIES AND THEIR APPLICATION IN PRACTICE

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Abstract : This on the subject inequalities concept , their types (linear , quadratic, irrational and others), solution methods and graphic to describe roads It is also studied how inequalities are expressed in real life . application , mathematical model creation also see the role will be released . Subject mathematician logic reinforcement and problems analysis to do skills to develop service does .

Key words: inequality , linear inequality , quadratic inequality , interval method , graph solution , mathematical model, solution package

Annotatsiya: Ushbu mavzu bo'yicha tengsizliklar tushunchasi, ularning turlari (chiziqli, kvadrat, irratsional va boshqalar), yechim usullari va yo'llarni tasvirlash uchun grafik. Shuningdek, tengsizliklar real hayotda qanday ifodalanishi o'rganiladi. Qo'llanilishi, matematik model yaratish, shuningdek, rolni ko'rish mumkin. Mavzu bo'yicha matematik mantiqni mustahkamlash va muammolarni tahlil qilish ko'nikmalarini rivojlantirish uchun xizmat qiladi.

Kalit so'zlar: Tengsizlik, chiziqli tengsizlik, kvadrat tengsizlik, interval usuli, grafik yechim, matematik model, yechimlar to'plami

Аннотация: В данной работе рассматриваются понятия неравенств, их типы (линейные, квадратичные, иррациональные) и другие, методы решения и графическое описание уравнений. Также изучается, как неравенства выражаются в реальной жизни. Применение и создание математических моделей, а также роль, которую они играют, будут раскрыты. Предметом исследования является закрепление математической логики и анализ задач для развития навыков обслуживания.

Ключевые слова: Неравенство, линейное неравенство, квадратичное неравенство, интервальный метод, графическое решение, математическая модель, пакет решений

Introduction

Mathematics is only theoretical concepts not in real life problems modeling and analysis to do for powerful In particular , **inequalities** daily in life decision acceptance to do , optimal resource distribution and security borders in marking wide is applied . Budget restrictions , maximum working release size , minimum

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energy expense such as many economic and technician issues inequalities through is expressed .

This in the article inequalities important from theorems was **AM-GM** and **Koshi-Bunyakovsky** inequalities deep analysis is done , their mathematician evidence will be brought both real and theoretical issues through application demonstration Also , the article of the students analytical thinking , proving and modeling skills to develop service does .

The main part

AM-GM inequality (Arithmetic-geometric average theorem)

Text : (you sent proofs based on is left , orderly as is written).

Generalized AM-GM or Koshi-Bunyakovsky inequality

Text : (you sent main proof stored , lemmas with), with your acquaintance possible:

Theorem (AM-GM): All positive $a_1, a_2, a_3, \dots a_n$ for numbers

$$\frac{a_1 + a_2 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n} \quad (1)$$

inequality appropriate . Equality $a_1 = a_2 = a_3 = \cdots = a_n$ is executed when.

Proof : By induction we use .

$$n = 2 \text{at} \sqrt{a_1 a_2} \leq \frac{a_1 + a_2}{2} \iff (\sqrt{a_1} - \sqrt{a_2})^2 \geq 0 \quad \forall a_1, a_2 \geq 0$$

(1) in We prove that it is n valid for ta positive numbers by assuming that ta is valid for $n + 1$ ta R^+ numbers. Let these numbers $a_1, a_2, a_3, \dots a_n, a_{n+1}$ be a_{n+1} the largest of them,

$$a_{n+1} \geq a_1, a_{n+1} \geq a_2, \dots a_{n+1} \geq a_n \implies$$

$$\implies a_{n+1} \geq \frac{a_1 + a_2 + \cdots + a_n}{n} \quad (2)$$

As follows designation we enter .

$$A_n = \frac{a_1 + a_2 + \cdots + a_n}{n} \quad ; \quad A_{n+1} = \frac{a_1 + a_2 + \cdots + a_n + a_{n+1}}{n} = \frac{nA_n + a_{n+1}}{n+1}$$

(2)to according to $a_{n+1} \geq A_n \implies a_{n+1} = A_n + \alpha \quad \alpha \geq 0$ Let's say. Then:

$$A_{n+1} = \frac{nA_n + A_n + \alpha}{n+1} = A_n + \frac{\alpha}{n+1}$$

This equality both part $n + 1$ Raising to the level, we find:

$$(A_{n+1})^{n+1} = A_n^{n+1} + C_{n+1}^1 (A_n)^n \frac{\alpha}{n+1} + \cdots + C_{n+1}^n (A_n) \left(\frac{\alpha}{n+1}\right)^n +$$

$$+ \left(\frac{\alpha}{n+1}\right)^{n+1} \geq (A_n)^{n+1} + C_{n+1}^1 (A_n)^n \frac{\alpha}{n+1}$$

$$= (A_n)^{n+1} + (A_n)^n \frac{\alpha}{n+1} \frac{(n+1)!}{1!\, n!}$$

$$= A_n^{n+1} + (A_n)^n \alpha = A_n^n (A_n + \alpha) = A_n^n \cdot a_{n+1}$$

To the assumption according to $A_n^n \geq a_1 a_2 \dots a_n \implies$

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$$(A_{n+1})^{n+1} \geq a_1 a_2 a_3 \dots a_n a_{n+1} \Rightarrow$$
$$\frac{a_1 + a_2 + \dots + a_n + a_{n+1}}{n+1} \geq \sqrt[n+1]{a_1 a_2 \dots a_n a_{n+1}}$$

From this come it turns out theorem proved .

Theorem (Generalized AM-GM or Cauchy-Bunyakovsky):

$a_1, a_2, \dots a_n, p_1, p_2, \dots, p_n$positive numbers

$$a_1^{p_1} a_2^{p_2} \dots . a_n^{p_n} \leq \left(\frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n}\right)^{p_1+p_2+\dots+p_n} \quad (1)$$

Relationship appropriate , equality and only $a_1 = a_2 = a_3 = \dots = a_n$is performed.

Proof : Lemma: $e^{x-1} \geq x$there is, in which $x \geq 1$.

Lemma The $\Rightarrow$ $e^{x-1} - x \geq e^{1-1} - 1 = 1 - 1 = 0 \Rightarrow$ $e^{x-1} - x \geq 0$proof is $f(x) \geq f(1)$that $\Rightarrow$ $x \geq 1$:$f(x) = e^{x-1} - x$

(1) in proof From the lemma we use .

$$S = \frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n}$$

to mark as we enter .

To the lemma according to : $e^{\frac{a_i}{S}-1} \geq \frac{a_i}{S}$it will be, in which case$i = \overline{1,n} \Rightarrow$ $S \cdot$

$$e^{\frac{a_i}{S}-1} \geq a_i \Rightarrow$$
$$a_i^{p_i} \leq S^{p_i} \cdot e^{\frac{a_i p_i}{S} - p_i}, \quad i = \overline{1,n}$$

These inequalities all of them multiply Let's go out .

$$a_1^{p_1} a_2^{p_2} \dots . a_n^{p_n} \leq \left(\frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n}\right)^{p_1+p_2+\dots+p_n}$$

Equality only $s = a_1 = a_2 = a_3 = \dots = a_n$is fulfilled, it follows that the inequality has been proven.

We are this from theorems using one how much following inequality about issues our solution possible .

Issues

Example 1: $x, y > 0$Prove the inequality if .$x^2 + y^2 + 1 \geq xy + x + y$

Proof :$x^2 + y^2 + 1 = \left(\frac{x^2}{2} + \frac{y^2}{2}\right) + \left(\frac{x^2}{2} + \frac{1}{2}\right) + \left(\frac{y^2}{2} + \frac{1}{2}\right) \geq xy + x + y$

From this come comes out inequality proved .

Example 2: $x > 0$if , $2^{\sqrt[12]{x}} + 2^{\sqrt[4]{x}} \geq 2 \cdot 2^{2\sqrt[6]{x}}$prove the inequality.

Proof : AM-GM theorem according to :$2^{\sqrt[12]{x}} + 2^{\sqrt[4]{x}} \geq 2 \cdot 2^{x^{\frac{1}{12}+\frac{1}{4}}} = 2 \cdot 2^{2x^{\frac{1}{6}}} = 2 \cdot 2^{2\sqrt[6]{x}}$

from this come comes out inequality proved .

Example $x^4 + y^4 + 8 \geq 8xy3$: $x, y > 0$If , what prove .

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Proof :$x^4+y^4+8=(x^4+4)+(y^4+4)\geq 2(2x^2+2y^2)\geq 2(4xy)=8xy$

⇨ Inequality proved .

Note : This is inequality. symmetrical of functions the most small whatever in determining is used .

Example 4:$x_1,x_2,\dots,x_5>0$ if the following prove :

$$x_1^2+x_2^2+x_3^2+x_4^2+x_5^2\geq x_1(x_2+x_3+x_4+x_5)$$

Proof :$\frac{x_1^2}{4}+x_2^2+\frac{x_1^2}{4}+x_3^2+\frac{x_1^2}{4}+x_4^2+\frac{x_1^2}{4}+x_5^2\geq x_1x_2+x_1x_3+x_1x_4+x_1x_5=$

$$=x_1(x_2+x_3+x_4+x_5)$$

⇨ Inequality proved .

Example 5:$x,y,z>0$ if so , then prove :

$$x^2+y^2+z^2-xy-yz-zx=$$

$$=\frac{x^2-2xy+y^2}{2}+\frac{y^2-2yz+z^2}{2}+\frac{z^2-2xz+x^2}{2}=$$

$$=\frac{(x-y)^2+(y-z)^2+(z-x)^2}{2}\geq 0$$

⇨ Inequality proved .

Example $(a+1)(b+1)(c+a)(b+c)\geq 16abc$6: $a,b,c>0$if , what prove .

Proof :$(a+1)(b+1)(c+a)(b+c)\geq 2\sqrt{a}\cdot 2\sqrt{b}\cdot 2\sqrt{ca}\cdot 2\sqrt{bc}=16abc$

⇨ Inequality proved .

Example $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 37$: $a,b,c>0$if , what prove :

Proof :$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3\sqrt[3]{\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{a}}=3$

⇨ Inequality proved .

Example 8: $a,b,c>0$If ,

$$\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}\leq\frac{a+b+c}{3}$$

what prove it .

Proof :$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geq 3\sqrt[3]{abc}\cdot\frac{3}{\sqrt[3]{abc}}=9$

⇨ Inequality proved .

Example 9: If$a,b,c>0,\ \ ab^2c^3=1$ if so , then prove :

$$\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\geq 6$$

Proof :$\frac{1}{a}+\frac{2}{b}+\frac{3}{c}=\frac{1}{a}+\frac{1}{b}+\frac{1}{b}+\frac{1}{c}+\frac{1}{c}+\frac{1}{c}\geq 6\sqrt[6]{\frac{1}{ab^2c^3}}=6$

⇨ Inequality proved .

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Example 10: If$a, b, c > 0$ if so , then prove it .

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}+4(\frac{ab+bc+ca}{a^2+b^2+c^2})^2 \geq 7$$

Proof : Cauchy-Bunyakovskii to the theorem according to :

$$(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2})(a^2b^2+b^2c^2+c^2a^2) \geq (a^2+b^2+c^2)^2$$

$$\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+4(\frac{ab+bc+ca}{a^2+b^2+c^2})^2 \geq 7$$

$$\frac{(a^2+b^2+c^2)^2}{3(a^2b^2+b^2c^2+c^2a^2)} \geq 1 \Rightarrow$$

$$1+\frac{2(a^2+b^2+c^2)}{3(a^2b^2+b^2c^2+c^2a^2)}+4(\frac{ab+bc+ca}{a^2+b^2+c^2})^2 \geq 7$$

$$\frac{1}{6}\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+\frac{1}{6}\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+\frac{1}{6}\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+$$

$$+\frac{1}{6}\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+2(\frac{ab+bc+ca}{a^2+b^2+c^2})^2+2(\frac{ab+bc+ca}{a^2+b^2+c^2})^2 \geq$$

$$6\cdot\sqrt[6]{\frac{1}{64}\cdot 2^4\cdot\left(\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}\right)^4} \geq 6 \Rightarrow$$

$$\Rightarrow \quad S \geq 6$$

⇨ Inequality proved .

Graphic and visualization

Python code (3D graphics for):

```
import matplotlib.pyplot as plt
import numpy as np
from mpl_ toolkits.mplot 3d import Axes3D

# x and y intervals designation
x = np.linspace ( 0.1 , 3 , 100 )
y = np.linspace ( 0.1 , 3 , 100 )
X, Y = np.meshgrid (x, y)

# Inequality both side
lhs = X** 2 + Y** 2 + 1          # left side : x^2 + y^2 + 1
rhs = X*Y + X + Y # right side : xy + x + y

# Graph drawing
fig = plt.figure ( figsize =( 12 , 6 ))
```


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```
    # Draw the left hand side (LHS)
ax1 = fig.add_ subplot ( 1 , 2 , 1 , projection= '3d' )
ax1.plot_surface ( X, Y, lhs , cmap = ' viridis ' , alpha= 0.9 )
ax1.set_title ( r'LHS : $x^2 + y^2 + 1$' )
ax1.set_xlabel( 'x' )
ax1.set_ylabel( 'y' )
ax1.set_zlabel( 'z' )

# Right draw the side (RHS)
ax2 = fig.add_ subplot ( 1 , 2 , 2 , projection= '3d' )
ax2.plot_ surface( X, Y, rhs , cmap = 'plasma' , alpha= 0.9 )
ax2.set_title ( r'RHS : $ xy + x + y$' )
ax2.set_xlabel( 'x' )
ax2.set_ylabel( 'y' )
ax2.set_zlabel( 'z' )

plt.tight_layout ( )
plt.show ()
```

Practical application (real-case examples)

In medicine

Medical in the protocols medicine dose the most less and the most many amounts usually **inequalities** through For example , the patient 's body weight looking at maximum permission done dose D≤k · mD \ leq k \ cdot m D ≤ k · m (this on the ground kk k — dose coefficient , mm m — weight) in the form This approach is defined as to the patient safe and individually tailored treatment method application opportunity gives .

In economics

Budget in planning every one resource expense general the budget not to exceed this is necessary and x1+x2+ ⋯ +xn≤Bx_1 + x_2 + \ dots + x_n \ leq B x1 + x2 + ⋯ + xn ≤ B in the form of **linear inequality** with Inequalities based on structured mathematician models optimal spending through strategies This is determined by the economic efficiency increase and limited from resources reasonable use provides .

In engineering

Facilities and technician systems constructive elements known downloads lift to receive this is necessary and endurance border For example , the stress value to the material typical maximum from the value not to exceed for s ≤ smax \ sigma \ leq \ sigma _{ max } s ≤ smax inequality is used . Such **inequalities engineering safety** and strength in guarantee important role plays .

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Conclusion

Inequalities mathematician analysis , engineering , economics and medicine such as many in the fields important tool as service This will do . in the article the most famous and strong inequality from theorems one was **AM-GM (arithmetic-geometric) average)** and **Koshi-Bunyakovsky** inequalities studied , their **induction** , **elementary algebra** and **exponent function based on** evidence was brought .

Quoted examples through this inequalities not only theoretically , maybe **practical** also showing **the** importance They were given . using complicated expressions assessment , problems simplification , as well as real -life restrictions as a mathematical model expression opportunity existence proved .

Also in the article inequalities :

in medicine – **dose borders in determining** ,

in economics – **budget distribution in planning** ,

in engineering – **mechanical security in providing**

how application shown .

In the future this the topic following in directions expansion to the goal appropriate will be :

Holder , **Chebyshev** and **Minkowski** inequalities such as progressive inequalities study ,

algorithmic analysis based on **computer through automated proof methods** use,

inequalities **at the Olympics** application analysis to do

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