OSILGAN MASSALARI BO‘LGAN DOIRAVIY QOVUSHOQ-ELASTIK PLASTINKALARNING ERKIN TEBRANISHLARI

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Annotatsiya. Doiraviy yupqa qovushoq-elastik plastinkalarga osilgan massalardan tashkil topgan mexanik sistemaning erkin tebranishlari qaralgan. Oxirgi yillarda radio elektron apparatlar (REA) korpuslarining materiallari steklotekstolit keng qo‘laniladi. Shuning uchun nazariy hisoblashlarda bu materiallardan foydalanilgan. Plastinka reologik xususiyatlarini hisobga olish chastotaning haqiqiy va mavhum qismlarini 10% ga kamaytirishga imkon berar ekan. Shunday qilib, biriktirilgan massalari bo‘lgan doiraviy plastinkaning dissipativ bir jinsli va bir jinsli bo‘lmagan mexanik tizimlarning erkin tebranishlari muammosini hal qilishning matematik qo‘yilishi va yechish metodikasi ishlab chiqilgan. Dissipativ mexanik tizim elementlari uchun kuchlanish va deformasiyalar orasidagi bog‘liqlik chiziqli irsiy Bolsman-Voltera munosabatlarini qanoatlantiradi. Chastotalar va dempferlash koeffisientining o‘lchovsiz chastota bikrlik koeffitsientiga bog‘liq o‘zgarishi keltirilgan.

Kalit so‘zlar: doiraviy yupqa qovushoq-elastik plastinka, osilgan massa, erkin tebranishlar, chastota, qovushqoqlik**.**

Abstract. Free oscillations of a mechanical system consisting of masses suspended from thin circular viscoelastic plates are considered. In recent years, the materials of radio-electronic devices (REA) housing, glass textolite, have been widely used. Therefore, these materials were used in theoretical calculations. Taking into account the rheological properties of the plate allows reducing the real and imaginary parts of the frequency by 10%. Thus, the mathematical formulation and methodology for solving the problem of free oscillations of dissipatively homogeneous and inhomogeneous mechanical systems of a circular plate with a combined mass have been developed. For elements of a dissipative mechanical system, the relationship between stresses and deformations satisfies the linear inherited Boltzmann-Volterra relations. The change in frequencies and the damping coefficient depending on the stiffness coefficient of the dimensionless frequency is presented.

Keywords: circular thin viscoelastic plate, suspended mass, free oscillations, frequency, viscosity.

Аннотация. Рассмотрены свободные колебания механической системы, состоящей из масс, подвешенных на круглых тонких вязкоупругих пластинах. В последние годы широко используются материалы корпусов радиоэлектронных аппаратов (РЭА) стеклотекстолит. Поэтому в

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теоретических расчетах использовались эти материалы. Учет реологических свойств пластинки позволяет снизить действительную и мнимую части частоты на 10%. Таким образом, разработана математическая постановка и методика решения задачи свободных колебаний диссипативно однородных и неоднородных механических систем круговой пластинки с присоединенными массами. Для элементов диссипативной механической системы зависимость между напряжениями и деформациями удовлетворяет линейному наследственному соотношению Больцмана-Вольтера. Приведено изменение частот и коэффициента демпфирования в зависимости от коэффициента жесткости безразмерной частоты.

Ключевые слова: круглая тонкая вязкоупругая пластина, подвешенная масса, свободные колебания, частота, вязкость.

1. Kirish

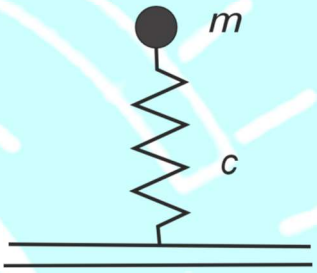
Harakatlanuvchi jismlarning konstruktiv elementlariga ta'sir etuvchi omillar yuzaga keladigan tebranish va zarba yuklari, akustik effektlar va boshqalar kiradi. Aerodinamik kuchlar harakatlanuvchi apparatlarning elementlari va qismlarining vibrasion tebranishlarini oshiradi. Bunday tuzilmalarni loyihalashda strukturaning xavfsizlik chegaralarini aniqlash, rezonans chastotalarini va strukturaning detallariga ta'sir qiluvchi faktorlarni hisoblash uchun dinamik hisob-kitoblarni amalga oshirish. Radio texnik apparatlarni hisoblash murakkablashadi, chunki bunday tuzilmalar juda ko‘p kichik qismlarga ega bo‘lgan murakkab shaklga ega elementlardan iborat. Mexanik sistemaning chiziqli elastik erkin va majburiy tebranishlarini hisoblash usullari analitik va sonli usullar yordamida to‘liqlikka yaqin qilingan. Ko‘plab adabiyotlar tebranishni amaliyotda qo‘llashga bag‘ishlangan bo‘lib, sonli hisoblashlar ham keng qo‘lanilgan. Xususan, elastik tebranish tizimlarini vibratsiyadan hisoblash nazariyasi adabiyotlarda batafsil yoritilgan. Erkinlik darajasi chekli bo‘lgan chiziqli elastik mexanik sistemada erkin chastotalar va tebranishlar formasini hisoblash muammosi standart dasturlardan foydalangan holda to‘liq hal qilingan. Tarqalgan parametrlarni elastik mexanik sistemalarni dinamik holatini hisoblash masalalari asosan sonli usullarni foydalanib yechilgan.

2. Metodlar

2.1. Masalaning qo‘yilishi va yechish usullari.

Yupqa deformasiyalanuvchi plastinkalarga yoki qobiqlarga biriktirilgan massalari bo‘lgan konstruksiyalar samolyot, uchuvchisiz uchuvchi apparatlar, ishlab chiqarish korxonalarida va qurilishda keng qo‘llaniladi. Qo‘yilgan masala massalar osilgan plastinkani erkin tebranishlarini topishga olib kelinadi. Bu yerda sistemalarda chastotalarni tekshirish rezonans hodisalalarini oldindan prognoz qilish imkonini beradi.

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1 - rasm. Plastinkaga o‘rnatilgan massa

Faraz qilaylik radusi R, qalinligi h, materialning zichligi ρ va operator formadagi silindrik bikirligi $\tilde{D}$ bo‘lgan qovushoq elastik plastinkaga $m_k (k=1,...,N)$ osilgan bo‘lsin. Operator bikirliklar quyidagicha

$$\tilde{D}f(t)=D_0\left[f(t)-\int_0^t R_D(t-\tau)f(\tau)d\tau\right];$$
$$\tilde{c}_\kappa f(t)=c_{0\kappa}\left[f(t)-\int_0^t R_{c\kappa}(t-\tau)f(\tau)d\tau\right] \quad (1)$$

Bu yerda $f(t)$– vaqtning ixtiyoriy funksiyasi, $R_D(t-\tau)$ va $R_{c\kappa}(t-\tau)$– relaksasiya yadrolari, $D_0=E_0h^3/(12(1-\nu^2))$, $c_{0\kappa}$ – oniy elastiklik moduli. U holda plastinka va unga osilgan massaning harakat- differensial tenglamalari quyidagicha bo‘ladi:

$$\tilde{D}\nabla^4 w+\rho h\ddot{w}=\sum_{k=1}^{N}\tilde{c}_k(y_k-w_k)\delta_k+q(r,\theta,t),$$
$$m_k\ddot{y}_k+\tilde{c}_k(y_k-w_k)=p_k(t), \quad (2)$$

Integro - differensial tenglamadan integralni yo‘qotish uchun Filatov va Sunchaliev tomonidan ishlab chiqilgan “muzlatish” usulini qo‘laymiz, u holda (1) quyidagi ko‘rinishni egalaydi

$$\bar{D}f(t)=D_0\left[1-\Gamma_D^C(\omega_R)-i\Gamma_D^S(\omega_R)\right]f(t)=D_0\Gamma_D,$$
$$\bar{c}_\kappa f(t)=c_{0\kappa}\left[1-\Gamma_{c\kappa}^C(\omega_R)-i\Gamma_{c\kappa}^S(\omega_R)\right]f(t)=c_{0\kappa}\Gamma_{ck}, \quad (3)$$

bu yerda

$$\Gamma_D^c(\omega_R)=\int_0^\infty R_D(\tau)\cos\omega_R\tau\,d\tau,\ \Gamma_{ck}^C(\omega_R)=\int_0^\infty R_{c\kappa}(\tau)\cos\omega_R\tau\,d\tau,$$
$$\Gamma_D^s(\omega_R)=\int_0^\infty R_D(\tau)\sin\omega_R\tau\,d\tau,\quad \Gamma_{ck}^s(\omega_R)=\int_0^\infty R_{c\kappa}(\tau)\sin\omega_R\tau\,d\tau.$$

U holda (2) differensial tenglama quyidagi ko‘rinishni egallaydi

$$\bar{D}\nabla^4 w+\rho h\ddot{w}=\sum_{k=1}^{N}\bar{c}_k(y_k-w_k)\delta_k+q(r,\theta,t),$$
$$m_k\ddot{y}_k+\bar{c}_k(y_k-w_k)=p_k(t). \quad (4)$$

Masalani yechishda chegaraviy va boshlangich shartlar qo‘yiladi

$$L_1(W)\big|_{r=R}=0, L_2(W)\big|_{r=R}=0, \quad (5)$$

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$$w|_{t=0}=0,\frac{\partial w}{\partial t}|_{t=0}=0,$$
$$y_k|_{t=0}=0,\frac{\partial y_k}{\partial t}|_{t=0}=0 \qquad (6)$$

Ixtiyoriy o‘zgarmas kattaliklarni topish uchun (5) shartdan foydalanamiz. Agar massa m_k lar osilgan nuqtalar (r_k,θ_k) ni ko‘chishlarini yozib chiqsak quyidagicha bo‘ladi

$$W_j(r_j,\theta_j)=\sum_{n=0}^{\infty}\left[A_nJ_n(\beta_br_j)+B_nI_n(\beta_br_j)\right]\cos(n\theta)-$$
$$-\sum_{k=1}^{N}\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\frac{\overline{c}_k(Y_k-W_k)V_{njk}}{D_0s_{nj}(\beta_b^2-\alpha_{1nj}^4)}V_{jik}(r,\theta),i=1,2,..N. \qquad (7)$$

Agar olingan yechimlarni chegaraviy shartlarga qo‘ysak $\overline{c}_k(Y_k-W_k)$ tenglamadan chiqarib olib soddalashtirsak $V_{nj}(r,\theta)$ o‘zgaruvchiga nisbatan quyidagicha algebraik tenglamalalar sistemasini olamiz.

$$A_nL_1\left[J_n(\beta_br)\right]\Big|_{r=R}+B_nL_1\left[I_n(\beta_br)\right]\Big|_{r=R}=0,$$
$$A_nL_2\left[J_n(\beta_br)\right]\Big|_{r=R}+B_nL_2\left[I_n(\beta_br)\right]\Big|_{r=R}=0,n=1,2..N. \qquad (8)$$

Bu holatda (8) dan $A_{ns}=B_{ns}=0$ bo‘ladi. Bu sistemaning (N-1)- tenglamasidan $Y_{js}=C_{js}Y_{Ns}$ (j=1,2…N;C_{Ns}=1).

Shunday qilib, olingan yechimlarga asoslanib tebranishlar formasini topamiz $(W_s(r,\theta)\,W_s(r,\theta))$

$$W_r(r,\theta)=-\frac{\beta_s^4}{\rho h}\sum_{k=1}^{N}C_{ks}m_k\sum_{n=0}^{\infty}\sum_{i=1}^{\infty}\frac{V_{nik}V_{nij}}{s_{ni}(\beta_s^4-\alpha_{1nj}^4)},V_{ks}=C_{ks}(C_{Ns}=1). \qquad (9)$$

Xos amplitudalar tenglamasini Y_{km} va Y_{kj} larni mos ravishda Y_{kj}, Y_{km} larga ko‘paytirib, ikinchi tenglamadan ayirsak va $Y_{mk}W_{kj}-Y_{kj}W_{km}$ ni tenglamadan chiqarib olsak, u holda $\omega_m\neq\omega_j$ ekanligidan ortogonalik shartini keltirib chiqaramiz

$$\int_0^R\int_0^{2\pi}W_mW_jrdrd\theta+\sum_{k=1}^{N}\frac{m_k}{\rho h}Y_{km}Y_{kj}=0,m\neq j\ . \qquad (10)$$

(2) tenglamaning umumiy yechimi quyidagicha bo‘ladi

$$w(r,\theta,t)=\sum_{m=1}^{\infty}q_m(t)W_m(r,\theta),y_k(t)=\sum_{m=1}^{\infty}q_m(t)Y_{km},k=1,...,N\ . \qquad (11)$$

Agar (11) yechimni (2) sistemaga qo‘yilsa, u holda qo‘yidagi sistemani olamiz

$$\sum_{m=1}^{\infty}\left(\ddot{q}_m+\omega^2q_m\right)\rho hW_m=q(r,\theta,t),$$
$$\sum_{m=1}^{\infty}\left(\ddot{q}_m+\omega^2q_m\right)m_kY_m=P_k(t),\ k=1,2,...N \qquad (12)$$

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Bu sistemani birinchi tenglamasini W_k ga ko‘paytirib, plastinka yuzasi bo‘yicha integrallasak, ikkinchi tenglamada k bo‘yicha yig‘sak va ortogonalik shartidan foydalansak, u holda quyidagi tenglamalarni olamiz

$$\ddot{q}_j + \omega_j^2 q_j = Q_j(t); j = 1,2... \quad (13)$$

bu yerda $Q_j(t)$ - umumlashgan kuch bo‘lib, quyidagi formula bilan aniqlanadi

$$Q_j(t) = \frac{1}{\rho h M_j}\left(\int_0^R \int_0^{2\pi} q(r,\theta,t) W_j(r,\theta) r dr d\theta + \sum_{k=1}^{n} P_k(t) Y_{kj}\right),$$

(12) tenglamani umumiy yechimi quyidagicha bo‘ladi

$$q_j(t) = A_j \cos\omega_j t + B_j \sin\omega_j t + \frac{1}{\omega_j}\int_0^t Q_j(\tau)\sin\omega_j(t-\tau)d\tau, \quad (14)$$

bu yerda A_j va B_j boshlang‘ich shartlardan topiladi:

$$\begin{aligned} &w(r,\theta,0) = w_0(r,\theta), \\ &\dot{w}(r,\theta,0) = \dot{w}_0(r,\theta), \\ &y_k\big|_{t=0} = y_{k0}, \\ &\dot{y}_k\big|_{t=0} = \dot{y}_{k0}, k = 1,2...N, \end{aligned} \quad (15)$$

va ortogonalik sharti quyidagi ko‘rinishni egallaydi

$$\begin{aligned} A_j &= \frac{1}{M_j}\left(\int_0^R \int_0^{2\pi} w_0 W_j r dr d\theta + \sum_{k=1}^{N} \frac{m_k}{\rho h} y_{k0}(t) Y_{kj}\right), \\ B_j &= \frac{1}{\omega_j M_j}\left(\int_0^R \int_0^{2\pi} w_0 W_j r dr d\theta + \sum_{k=1}^{N} \frac{m_k}{\rho h} \dot{y}_{k0}(t) Y_{kj}\right), \end{aligned} \quad (16)$$

Tizim elementlarining qovushqoqlik xususiyatlarini yoki nuqtaviy ulanishlarida sodir bo‘ladigan deformasiyalanish jarayonlarini tavsiflash uchun Bolsman-Volter chiziqli irsiy nazariyasidan foydalanamiz:

$$\sigma_{mk}^n(t) = E_n\left[\varepsilon_{mk}^n(t) - \int_{-\infty}^{t} R^n(t-\tau)\cdot\varepsilon_{mk}^n(\tau)\, d\tau\right], \quad (17)$$

Kuchlanishlar vaqtning davriy funksiyasi bo‘lishi uchun (17) irsiy integralning pastki chegarasi minus cheksizlikka teng deb qabul qilinadi. Agar pastki chegara nolga teng bo‘lsa, kuchlanish vaqt o‘tishi bilan pasayib ketadigan aperiodik tebranishlarni beradi.

Relasatsiya yadrosi $R(t-\tau)$ integrallanuvchanlik, uzluksizlik ($t = \tau$ dan tashqaridan), musbat aniqlanganlik va monotonlik kabi xossalarga ega bo‘lishi kerak:

3. Sonli natijalar va ularning tahlili

Yuqorida olingan analitik yechim asosida Gauss, Laplos teoremasi, Maxsus funksiyalaoni hisoblash va Myuller usulari asosida yaratilgan (C^{++}) dastur bilan sonli natijalar olindi. Yular massasi M_1=0.1M_0, bu yerda M_0 пластинка массаси.

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Пластинкага осилган масса нуқтавий деб олинган. Пластинканинg хос тебранишлари ўрганилган. У ҳолда хос частоталар топилади. Пластинканинг физик - механик характеристикаси қуйидагича $E = 2.1 \cdot 10^{11} Па; \rho = 7.85 \cdot 10^3 кг/м^3, \nu = 0.3$, радиуси R ва қалинлиги 0.5х0.001м, relaksasiya yadrosining parametrlari quyidagicha: $A = 0{,}048;\ \beta = 0{,}05;\ \alpha = 0{,}1$.

1- Jadval.

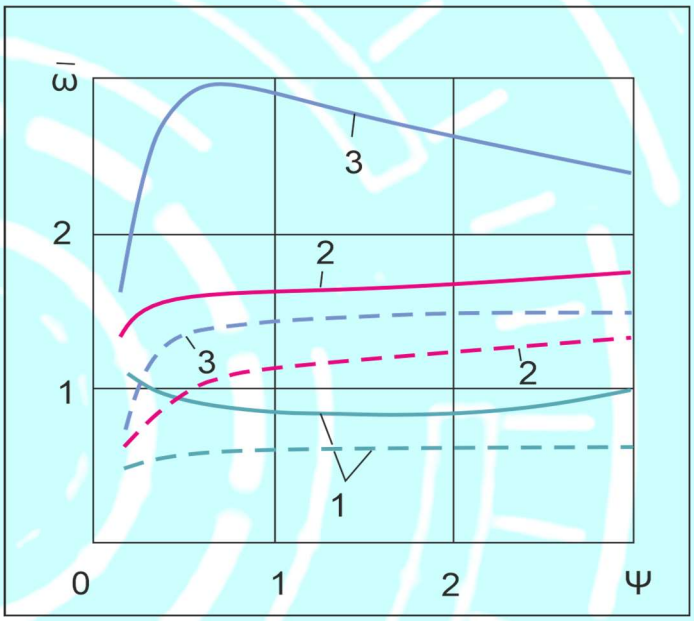
Plastinkaning egilishdagi chastotasini [51] ish bilan solishtirish

Plastinka o‘lchamlari h/R	Dissertasiyada olingan sonli natijalar	[11] ishda olingan sonli natijalar, Гц.	Xatolik, %
0,20	1071,17	1079,00	0,14
0,40	1106,32	1197,18	3,12
0.60	1273,24	1265,78	3,35
1.0	1301,43	1307,86	4,07
2.0	1567,32	1563,1	4,65

Ko‘rinib trubdiki, olinadigan natijalar kichik chastotalarda 6% gacha xatolik berar ekan.

Oxirgi yillarda radio elektron apparatlar korpuslarining materiallari steklotekstolit keng qo‘laniladi. Shuning uchun nazariy hisoblashlarda bu materiallardan foydalanilgan. Bu materialni fizik- mexanik xususiyatlari quyidagicha: – Yung moduli (E) -10^5 кгс/м^2, – zichligi (ρ), - 1400 кг/м^3, – chastota (Ω), - 200. Tashqi vibratsion nagruzka amplitusini 1 mm deb olamiz. Vibratsion ko‘chish ta'sirida mexanik sistema tebransa, u holda $u_0(t) = A_0 \sin(\Omega t + \varphi)$ ko‘rinishda olinadi. Bu yerda A_0 - viratsion yuklanish aplitudasi, Ω - tashqi vibratsion yuklanishning davriy chastotasi, φ - boshlang‘ich faza. a) qattiq mahkamlannganlik, б) sharnirli tayanch shartlari uchun olindi. Natijalar 2.2-rasmda keltirilgan.

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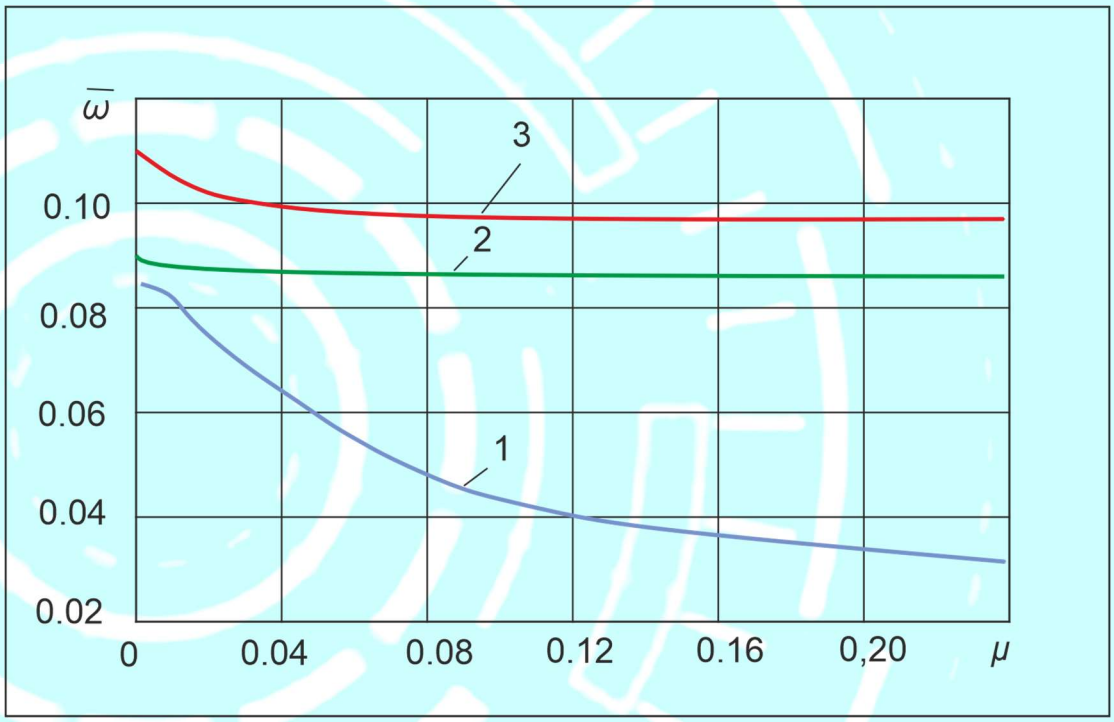


2 - rasm. Chastotani plastinka qalinligi bo‘yicha o‘zgarishi keltirilgan.

Bu 2.2 - rasmda asosiy chastotani plastinka qalinligi bo‘yicha o‘zgarishi keltirilgan. Tebranishlar chastotasini birinchi va ikinchi modasini prujinkaning bikirligiga bog‘liq o‘zgarishi keltirilgan. Dissipativ bir jinsli bo‘lmagan mexanik sistema uchun oldingi masalada olingan mexanik samara bu yerda ham o‘rinli bo‘lishi topildi. Yig‘ilgan massalar sonini ko‘payishi erkin tebranishlar chastotasini haqiqiy va mavhum qismini 20% kamaytirishga olib kelar ekan.

Osiladigan massalar sonini oshishi chastotalarni kamayishiga olib kelar ekan. Chastotalarni mavhum qismini o‘zgarishi ham real chastotani o‘zgarish tendensiyasiga yaqin bo‘lar ekan. Keyingi 3 - rasmda 2 ta simetirik massa osilganda steklotekstolit material uchun osilgan massani plastinka massasiga nisbati olingan (sharnirli plastinka uchun) . Rasmda uchta chastotalar modasini massalar nisbatiga bog‘liq o‘zgarishi keltirilgan.

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3 – rasm. Chastota haqiqiy qismini 3 ta modasini massalar nisbatiga bog‘liq o‘zgarishi

2- jadval.

Xos chastotaning beshta modasini k ga bog‘liq o‘zgarishi

k	ω_1	ω_2	ω_3	ω_4	ω_5
1	0.08237	0.23371	0.42371	1.90235	2.52379
2	0.07143	0.23113	0.42143	0.92143	1.92353
3	0.06152	0.21628	0.43201	0.87152	1.91452
4	0.05474	0.12316	0.41414	0.76014	0.96147
5	0.04139	0.11432	0.41101	0.71392	0.84132
6	0.03127	0.11326	0.33801	0.71276	0.82278
7	0.02128	0.11248	0.31501	0.71128	0.81027

Xos chastotaning beshta modasini k ga bog‘liq o‘zgarishi 2-jadvalga keltirilgan. Plastinka reologik xususiyatlarini hisobga olish chastotani haqiqiy va mavhum qismini 10% kamaytirishga imkon berar ekan. Shunday qilib biriktirilgan massalari bo‘lgan doiraviy plastinkaning dissipativ bir jinsli va bir jinsli bo‘lmagan mexanik tizimlarning erkin tebranishlari muammosini hal qilishning matematik qo‘yilishi va yechish metodikasi ishlab chiqilgan. Dissipativ mexanik tizim elementlari uchun kuchlanish va deformasiyalar orasidagi bog‘liqlik chiziqli irsiy Bolsman-Voltera munosabatlarini qanoatlantiradi.

Xulosalar

1. Nuqtali tayanchlari va biriktirilgan massalari bo‘lgan plastinkalar iborat dissipativ bir xil va bir jinsli bo‘lmagan mexanik tizimlarning erkin va majburiy tebranishlari muammosini matematik qo‘yilishi, yechish metodikasi va algoritimi

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ishlab chiqilgan. Ishlab chiqilgan algoritimning ishonchliligi sonli tajtiba asosida isbotlangan;

2. Tayanchlar va yig‘ilgan massasi bo‘lgan dissipativ bir jinsli (va bir jinsli bo‘lmagan) qobiqlar tizimlarining (erkin va majburiy) tebranishlari muammolari hal qilinadi. Butun tizimning dissipativ xususiyatlarini tavsiflash uchun I.E . Troyanovskiy va I.I. Safarov tomonidan kiritilgan global so‘nish (demfirlash) koeffisienti (GSK) tushunchasidan foydalanildi.

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