IKKI KOMPONENTALI MUHITDA PARABOLIK TENGLAMALAR SISTEMASINI SONLI YECHIMI

Fayzullayeva Zarnigor Inatillayevna
Muhammad al-Xorazmiy nomidagi TATU, Dasturlash texnologiyalari kafedrasi dotesnti.
E-mail: Zarnigor18z02@gmail.com

Annotatsiya: Tabiatda uchraydigan ko‘plab fizik, kimyoviy, biologik va ekologik jarayonlar o‘z tabiatiga ko‘ra nochiziqli bo‘lib, ularni tavsiflashda nochiziqli differensial tenglamalar va ularning sistemalaridan keng foydalaniladi. Bunday jarayonlar ko‘pincha kvazichiziqli yoki to‘liq nochiziqli tenglamalar ko‘rinishida namoyon bo‘ladi va ularga mos ravishda boshlang‘ich hamda chegara shartlari berilgan holda matematik masala shaklida tadqiq qilinadi. Hozirgi zamonaviy ilm-fanning asosiy matematik modellarining katta qismi chiziqli bo‘lmagan qisman differensial tenglamalar bilan ifodalanishi tufayli ularni tahlil qilish va amaliy masalalarda qo‘llash samarali sonli yechimlarni ishlab chiqishni talab qiladi. Mazkur ishda nochiziqli differensial tenglamalar hamda ularning sistemalariga tegishli masalalarni sonli yechish bo‘yicha yondashuvlar ko‘rib chiqiladi. Xususan, murakkab fizik jarayonlarni ifodalovchi nochiziqli modellarni hisoblashda iterativ usullar, barqaror sxemalar, differensial operatorlarni diskretlashtirish va kompyuterda implementatsiya qilish masalalari o‘rganiladi. Taklif etilgan sonli usullar yordamida nochiziqli modellarni aniq va barqaror yechish imkoniyatlari ko‘rsatilib, ular real jarayonlarni modellashtirishda samarali ekanligi asoslab beriladi.

Kalit so‘zlar: dempfirlash, konvektiv ko‘chish, balans usuli, ayirmali sxema, algebraik tenglamalar sistemasi

Abstract: Many physical, chemical, biological, and ecological processes observed in nature are inherently nonlinear, and their behavior is often described using nonlinear differential equations and systems of such equations. These processes typically appear in the form of quasilinear or fully nonlinear equations and are studied as mathematical problems with corresponding initial and boundary conditions. Since most modern scientific models are represented by nonlinear partial differential equations, their analysis and practical application require the development of efficient numerical solution methods. This study examines numerical approaches for solving problems related to nonlinear differential equations and their systems. In particular, it investigates iterative techniques, stable numerical schemes, discretization of differential operators, and the implementation of these methods in computational environments for modeling complex physical phenomena. The proposed numerical methods are shown to provide accurate and

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stable solutions to nonlinear models, demonstrating their effectiveness in simulating real-world processes.

Аннотация: Многие физические, химические, биологические и экологические процессы, наблюдаемые в природе, по своей сути являются нелинейными, поэтому для их описания широко используются нелинейные дифференциальные уравнения и системы таких уравнений. Эти процессы, как правило, проявляются в виде квазилинейных или полностью нелинейных уравнений и исследуются как математические задачи с соответствующими начальными и граничными условиями. Поскольку большинство современных научных моделей выражаются нелинейными уравнениями в частных производных, их анализ и практическое применение требуют разработки эффективных методов численного решения. В данной работе рассматриваются численные подходы к решению задач, связанных с нелинейными дифференциальными уравнениями и их системами. Особое внимание уделено итерационным методам, устойчивым численным схемам, дискретизации дифференциальных операторов и компьютерной реализации этих методов при моделировании сложных физических процессов. Показано, что предложенные численные методы обеспечивают точное и устойчивое решение нелинейных моделей, что подтверждает их эффективность при моделировании реальных процессов.

Kirish

Tabiatda ro‘y beradigan ko‘plab fizik, kimyoviy, biologik, ekologik va texnologik jarayonlar o‘z mohiyatiga ko‘ra murakkab va ko‘p omilli bo‘lib, ularning aksariyati nochiziqli xususiyatlarga ega. Bunday jarayonlarda moddaning harakati, energiyaning taqsimlanishi, konsentratsiyaning o‘zgarishi yoki fizik-kimyoviy reaksiyalarning kechishi oddiy chiziqli qonunlar bilan izohlab bo‘lmaydi. Shu sababli zamonaviy matematika va amaliy fizika fanlarida nochiziqli differensial tenglamalar va ularning sistemalari tabiiy jarayonlarni modellashtirishning asosiy vositasiga aylanib bormoqda.

Nochiziqli modellar bugungi ilm-fan taraqqiyotining barcha sohalarida — jumladan, issiqlik va massa almashinuvi, suyuqliklar dinamikasi, kimyoviy reaksiyalar kinematikasi, populyasion biologiya, ekologik muvozanat, geofizik jarayonlar hamda texnologik jarayonlarni boshqarish kabi yo‘nalishlarda keng qo‘llaniladi. Bu kabi modellar odatda boshlang‘ich va chegara shartlari bilan birga qaraladi va ularning analitik yechimlarini topish ko‘pincha murakkab yoki butunlay imkonsiz bo‘ladi. Shu bois, nochiziqli tenglamalar va tenglamalar sistemasini tahlil qilishda sonli usullarni ishlab chiqish va takomillashtirish muhim ilmiy vazifalardan biridir.

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Hozirgi kunda nochiziqli differensial tenglamalarning sonli yechimiga doir ilmiy tadqiqotlar ko‘plab yondashuvlarni taklif etmoqda: iterativ usullar, barqaror va aniq farqlanish sxemalari, differensial operatorlarning diskretlashtirilgan modellarini yaratish, hisoblash algoritmlarini optimallashtirish va ularni dasturiy ta’minot ko‘rinishida amalga oshirish shular jumlasidandir. Bunday usullar orqali real jarayonlarning xulq-atvori, ularning barqarorligi, tarqalish tezligi, fazaviy lokalizatsiya kabi xususiyatlarini chuqur o‘rganish imkoniyati yaratiladi.

Mazkur ish nochiziqli differensial tenglamalar va ularning sistemalariga tegishli masalalarni sonli yechish, barqaror va ishonchli hisoblash algoritmlarini qurish hamda ularni amaliy jarayonlarga tadbiq etish masalalarini qamrab oladi. Tadqiqot natijalari real fizik jarayonlarni modellashtirishda qo‘llaniladigan sonli yondashuvlarning samaradorligini asoslashga qaratilgan bo‘lib, murakkab nochiziqli modellarni amaliyotga tatbiq etishda muhim ilmiy-amaliy ahamiyat kasb etadi.

Muallaiflarining ishlarida va ulardagi havolalarda ko‘rsatilgandek, avtomodel tenglamalar nochiziqli qisman differensial tenglamalar yechimlarining sifat xususiyatlarini o‘rganishda muhim rol o‘ynaydi, chunki yechimlarni avtomodel yechim qurish yordamida yechish yangi nochiziqli hodisalarning xarakterli xususiyatlarini aniqlash mumkin[1,2].

Ishda sonli sxemalar va $(k=1,\ p=2,\ m>1)$ bo‘lganda, g‘ovakli muhit tenglamalari uchun taklif qilingan avtomodel yechim qurush usuli asosida haydash usuli taklif qilingan[2].

Masalaning $u(t,x)$ yechimi shundan iboratki, $\forall t>0$ uchun $l(t)>0$ funksiya mavjud bo‘lib, $u(t,x)\equiv 0\ |x|\in(l(t),+\infty)$ da unda buzilish tarqalishining cheklangan tezligi mavjud[3].

Muhokama va natijalar

Quyidagi konvektiv ko‘chishga ega bo‘lgan va dempfirlash ta’sirida ikki karra nochiziqli diffuziya tenglamalar sistemasini $Q=\{(t,x):t>0,x\in R\}$ da qaraymiz.

$$\begin{cases}\dfrac{\partial u}{\partial t}=\dfrac{\partial}{\partial x}\left(\left|\dfrac{\partial u^{k_1}}{\partial x}\right|^{p-2}\dfrac{\partial u^{m_1}}{\partial x}\right)\pm s(t)\dfrac{\partial u}{\partial x}-b_1u^{q_1}\left|\dfrac{\partial v^{m_1}}{\partial x}\right|^{p_1},\\ \dfrac{\partial v}{\partial t}=\dfrac{\partial}{\partial x}\left(\left|\dfrac{\partial v^{k_2}}{\partial x}\right|^{p-2}\dfrac{\partial v^{m_2}}{\partial x}\right)\pm s(t)\dfrac{\partial v}{\partial x}-b_1v^{q_2}\left|\dfrac{\partial u^{m_2}}{\partial x}\right|^{p_2}.\end{cases} \qquad (3.3.1)$$

$$u(x,0)=u_0(x)\geq 0,\ \ v(x,0)=v_0(x)\geq 0,\ \ x\in R^1 \qquad (3.3.2)$$

3.3.1 ni quyidagicha qayta yozib olamiz

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$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(B_1(u)\dfrac{\partial u}{\partial x}\right) + c\upsilon_1(t)\dfrac{\partial u}{\partial x} - b_1 u^{q_1}\left|\dfrac{\partial v^{m_1}}{\partial x}\right|^{p_1}, \\ \dfrac{\partial v}{\partial t} = \dfrac{\partial}{\partial x}\left(B_2(v)\dfrac{\partial v}{\partial x}\right) + c\upsilon_2(t)\dfrac{\partial u}{\partial x} - b_2 v^{q_1}\left|\dfrac{\partial u^{m_2}}{\partial x}\right|^{p_2}, \end{cases} \tag{3.3.3}$$

$$B_1(u) = m_1\left|k_1 u^{k_1}\frac{\partial u}{\partial x}\right|^{p-2} u^{m_1-1},$$

$$B_2(v) = m_2\left|k_2 v^{k_2-1}\frac{\partial v}{\partial x}\right|^{p-2} v^{m_2-1}.$$

(3.3.1) va (3.3.2) masalani quyidagi to‘rda chekli ayirmalar sxemasiga o‘tamiz

Q sohada to‘r quramiz [108; 238–248-b.] $\overline{\omega}_h$ x bo‘yicha h qadam bilan:

$$\omega_h = \{x_i = ih, \quad h>0, \quad i = 0,1,...,n, \quad hn = b\},$$

va vaqt bo‘yicha quidagi to‘rni olamiz

$$\omega_\tau = \{t_j = j\tau, \quad \tau>0, \quad j = 0,1,...,m, \quad \tau m = T\}.$$

va quyidagi boshlang‘ich va chegaraviy shartlar bilan sonli yechish sxemasini quramiz.

$$\begin{cases} u(t,0) = \phi_1(t) > 0, \ u(t,b) = \phi_2(t) = 0, \ t\in[0,T]; \\ v(t,b) = \phi_3(t) = 0, \ v(t,b) = \phi_4(t) = 0, \ t\in[0,T]; \\ u(x,0) = u_0(x), \qquad v(x,0) = u_0(x), \qquad x\in R_+; \end{cases}$$

Balans usulini qo‘llagan holda (3.3.1) masalani oshkormas ayirmali sxema bilan almashtiramiz va ayirmali masalaga ega bo‘lamiz:

$$\begin{cases} \dfrac{y_i^{j+1}-y_i^j}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}(y)\left(y_{i+1}^{j+1}-y_i^{j+1}\right) - a_i(y)\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right) + c_1\upsilon_1(t)\dfrac{y_i^{j+1}-y_{i-1}^{j+1}}{h} - \\ -G_1\left(y_i^j, g_i^j\right), \\ \dfrac{g_i^{j+1}-g_i^j}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}(g)\left(g_{i+1}^{j+1}-g_i^{j+1}\right) - a_i(g)\left(g_i^{j+1}-g_{i-1}^{j+1}\right)\right) + c_2\upsilon_2(t)\dfrac{g_i^{j+1}-g_{i-1}^{j+1}}{h} - \\ -G_2\left(y_i^j, g_i^j\right), \end{cases}$$

(3.3.3)

$$G_1\left(y_i^j, g_i^j\right) = b_1\left(y_i^j\right)^{q_1}\left|m_1\left(g_i^j\right)^{m_1-1}\frac{g_i^j - g_{i-1}^j}{h}\right|^{p_1},$$

$$G_2\left(y_i^j, g_i^j\right) = b_1\left(g_i^j\right)^{q_2}\left|m_2\left(y_i^j\right)^{m_2-1}\frac{y_i^j - y_{i-1}^j}{h}\right|^{p_2},$$

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bu yerda $a_i(y)$ va $a_i(g), i=1,2$ larni hisoblash uchun quyidagi formulalardan biri qo‘llaniladi

$$\begin{cases} a_1(y) = B_1\left(\dfrac{y_i^j - y_{i-1}^j}{2}\right) \\ a_2(g) = B_2\left(\dfrac{g_i^j - g_{i-1}^j}{2}\right) \end{cases} \tag{3.3.4}$$

$$\begin{cases} a_i(y) = \dfrac{B_1(y_i^j) + B_1(y_{i-1}^j)}{2} \\ a_i(g) = \dfrac{B_2(g_i^j) + B_2(g_{i-1}^j)}{2} \end{cases} \tag{3.3.5}$$

(3.3.1) tenglamalar sistemasini yechish uchun oddiy iteratsiya usulidan foydalanamiz va quyidagini hosil qilamiz:

$$\begin{cases} \dfrac{y_i^{j+1,s+1} - y_i^{j,s}}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}^{s}(y)\left(y_{i+1}^{j+1,s+1} - y_i^{j+1,s+1}\right) - a_i^{s}(y)\left(y_i^{j+1,s+1} - y_{i-1}^{j+1,s+1}\right)\right) + \\ + s(t)\dfrac{y_i^{j+1,s+1} - y_{i-1}^{j+1,s+1}}{h} - G_1\left(y_i^{j,s}, g_i^{j,s}\right) \\ \dfrac{g_i^{j+1,s+1} - g_i^{j,s}}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}^{s}(g)\left(g_{i+1}^{j+1,s+1} - g_i^{j+1,s+1}\right) - a_i^{s}(g)\left(g_i^{j+1,s+1} - g_{i-1}^{j+1,s+1}\right)\right) + \\ + s(t)\dfrac{g_i^{j+1,s+1} - g_{i-1}^{j+1,s+1}}{h} - G_2\left(y_i^{j,s}, g_i^{j,s}\right) \end{cases} \tag{3.3.6}$$

bunda $s = 0,1,2,\ldots.$

$$\begin{cases} G_1\left(u_i^{j,s}, v_i^{j,s}\right) = b_1\left(u_i^j\right)^{q_1}\left|\dfrac{(v_{i+1}^{j+1})^m - (v_i^{j+1})^m}{h}\right|^{p_1}, \\ G_2\left(u_i^{j,s}, v_i^{j,s}\right) = b_1\left(v_i^j\right)^{q_2}\left|\dfrac{(u_{i+1}^{j+1})^m - (u_i^{j+1})^m}{h}\right|^{p_2}, \end{cases}$$

y_i^{s+1} va g_i^{s+1} ga nisbatan (3.3.6) ayirmali sxemalar chiziqli. y va g ning qiymati, y_i^{s+1}, g_i^{s+1} uchun boshlang‘ich iteratsiya sifatida vaqt bo‘yicha avvalgi qadamdan olinadi: $y^{j+1,0} = y^j$, $g^{j+1,0} = g^j$. Iteratsion sxema bo‘yicha hisoblashda iteratsiya aniqligi beriladi va jarayon quyidagi shartlar qanoatlantirilguncha davom etadi

$$\max_{0\le i\le n}\left|y_i^{s+1} - y_i^{s}\right| < \varepsilon, \quad \max_{0\le i\le n}\left|g_i^{s+1} - g_i^{s}\right| < \varepsilon.$$

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(3.3.6) ayirmali tenglamalar sistemasiga quyidagi belgilashlarni kiritamiz:

$$A_{1i}=\frac{\tau}{h^2}a_{i+1}\left(\overset{s}{y}\right)y_{i+1}^{j+1},\qquad A_{2i}=\frac{\tau}{h^2}a_{i+1}\left(\overset{s}{g}\right)g_{i+1}^{j+1}$$

$$B_{1i}=\left(\frac{\tau}{h^2}a_i\left(\overset{s}{y}\right)-\frac{\tau c}{h}\upsilon\left(t_j\right)\right)y_{i+1}^{j+1},\qquad B_{2i}=\left(\frac{\tau}{h^2}a_i\left(\overset{s}{g}\right)-\frac{\tau c}{h}\upsilon\left(t_j\right)\right)g_{i+1}^{j+1},$$

$$C_{1i}=A_{1i}+B_{1i}+1-\frac{c\upsilon_1\left(\overset{s}{t}\right)}{h},\qquad C_{2i}=A_{2i}+B_{2i}+1-\frac{c\upsilon_2\left(\overset{s}{t}\right)}{h},$$

$$F_{1i}=\overset{s}{y}_i^j-\tau G_1\left(\overset{s}{y}_i^j,\overset{s}{g}_i^j\right),\qquad F_{2i}=\overset{s}{g}_i^j-\tau G_2\left(\overset{s}{y}_i^j,\overset{s}{g}_i^j\right),$$

natijada (3.3.6) sistema quyidagi ko'rinishga keladi:

$$\begin{cases}A_{1i}\overset{s+1}{y}_{i+1}^{j+1}-C_{1i}\overset{s+1}{y}_i^{j+1}+B_{1i}\overset{s+1}{y}_{i-1}^{j+1}=-F_{1i},\ \ i=1,2,...,n-1,\\ A_{2i}\overset{s+1}{g}_{i+1}^{j+1}-C_{2i}\overset{s+1}{g}_i^{j+1}+B_{2i}\overset{s+1}{g}_{i-1}^{j+1}=-F_{2i},\ \ i=1,2,...,n-1,\end{cases}\qquad(3.3.7)$$

(3.3.7) algebraik tenglamalar sistemasi yechish uchun haydash (progonka) usulini qo'llaymiz. Ushbu metodga asosan

$$\begin{cases}\overline{y}_i=\alpha_{1i}\left(\beta_{1i}+\overline{y}_{i+1}\right),\\ \overline{g}_i=\alpha_{2i}\left(\beta_{2i}+\overline{g}_{i+1}\right),\end{cases}\qquad(3.3.8)$$

bu yerda α_{1i}, α_{2i}, β_{1i}, β_{2i} - koeffitsientlar quyidagicha hisoblanadi:

$$\begin{cases}\alpha_{1i+1}=\dfrac{B_{1i}}{C_{1i}-\alpha_{1i}A_{1i}},\\ \alpha_{2i+1}=\dfrac{B_{2i}}{C_{2i}-\alpha_{2i}A_{2i}},\end{cases}$$

$$\begin{cases}\beta_{1i+1}=\dfrac{A_{1i}\beta_{1i}+F_{1i}}{C_{1i}-\alpha_{1i}A_{1i}},\\ \beta_{2i+1}=\dfrac{A_{2i}\beta_{2i}+F_{2i}}{C_{2i}-\alpha_{2i}A_{2i}},\end{cases}$$

bunda $i=1,2,\ldots n$. Boshlang'ich α_{10}, α_{20}, β_{10}, β_{20} qiymatlar yuqorida berilgan chegaraviy shartlardan topiladi.

Progonka usulini qo'llash uchun yetarli shart diagonal ustunlik shartidir.

$$A_i\neq 0, B_i\neq 0, \left|A_i\right|+\left|B_i\right|\le C_i$$

Yuqoridagi ayirmali sxemalardan foydalanib, hisoblash tajribasi o'tkazildi. Raqamli tajribalar natijalari h=0.01 yetarlicha ayirmali sxema bosqichida olinadi va iteratsiyaning $\varepsilon=10^{-3}$ aniqligi aniqlanadi.

Xulosa

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Konvektiv ko‘chish va dempfirlash hadiga ega bo‘lgan nochiziqli diffuziya jarayoni uchun Koshi masalasi tadqiq etildi. Buning uchun avtomodel va taqribiy avtomodel yondoshuv asosida avtomodel yechim qurib olindi. Shuningdek avtomodel yechim asimtotikasi o‘rganildi. O‘rganilgan matematik modelni bir jinsli bo‘lmagan muhitda diffuziya jarayoni tavsiflovchi, fazoviy lokalizatsiya effekti olindi, diffuziya jarayonini borish tezligi chekliligi o‘rganildi va qurib olingan yechimning finit xossasi o‘rganildi. Yechimlarning sifat xossalariga asoslanib, sonli tajriba sinov natijalari olindi. Hisoblash tajribalari natijalari shuni ko‘rsatadiki, qo‘llanilgan sonli usullar o‘rganilayotgan nochiziqli masalalarni yechishda samarali bo‘ldi.

Konvektiv ko‘chish va dempfirlash hadiga ega bo‘lgan nochiziqli tenglamalar sistemasi uchun Koshi masalasini n=1 holi uchun sonli yechishda C# dasturlash tilida dasturiy ta’minot yaratildi va vizuallashtirish JavaSriptda amalga oshirildi. Tajriba sinov natijalari tenglamalar sistemasining parametrlarining turli qiymatlarida olindi. Pragonka usulidan foydalangan holda oshkormas sxema qo‘llaniladi. Iterativ jarayonda chiziqli ko‘rinishga keltirish uchun Maxsus usulidan foydalanildi.

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