ZICHLIK VA MANBA EGA NOCHIZIQLIK TENGLAMALAR SISTEMASINI SONLI YECHISH ALGORITMI

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Annotatsiya. Ushbu ish nochiziqli parabolik tipdagi differensial tenglamalar sistemasi uchun Koshi masalasini matematik modellashtirish va sonli yechish masalalariga bag‘ishlangan. Modelda vaqt va fazoga bog‘liq bo‘lgan manbali hadlar ishtirok etib, zichlik va tezlik komponentlarining evolyutsiyasi tavsiflanadi. Masalani yechishda avtomodel yondashuv asosida boshlang‘ich yaqinlashish qurilib, hisoblash jarayonining barqarorligi va yaqinlashuvini ta’minlovchi oshkormas ayirmali sxemalar qo‘llanilgan. Nochiziqli algebraik tenglamalar sistemalarini yechish uchun iteratsiya usullari hamda haydash algoritmidan foydalanilgan. Ko‘p o‘lchovli hol uchun Pismen–Rechford sxemasiga asoslangan o‘zgaruvchan yo‘nalishlar usuli tatbiq etilgan. Sonli tajribalar natijalari taklif etilgan algoritmlarning samaradorligi va ishonchliligini tasdiqlaydi.

Kalit so‘zlar: matematik modellashtirish, nochiziqli tenglamalar, Koshi masalasi, sonli usullar, oshkormas sxema, avtomodel yondashuv.

Abstract. This paper is devoted to the mathematical modeling and numerical solution of the Cauchy problem for a system of nonlinear parabolic-type differential equations. The model involves source terms dependent on time and space and describes the evolution of density and velocity components. To solve the problem, an initial approximation is constructed based on the self-similar (automodel) approach, and implicit finite difference schemes are employed to ensure stability and convergence of the computational process. Iterative methods and the Thomas algorithm are used to solve the resulting nonlinear algebraic systems. For the multidimensional case, an alternating direction method based on the Peaceman–Rachford scheme is applied. Numerical experiments confirm the efficiency and reliability of the proposed algorithms.

Keywords: mathematical modeling, nonlinear equations, Cauchy problem, numerical methods, implicit scheme, self-similar approach.

Аннотация. Данная работа посвящена математическому моделированию и численному решению задачи Коши для системы нелинейных дифференциальных уравнений параболического типа. В модели учитываются источниковые члены, зависящие от времени и пространства, и описывается эволюция компонент плотности и скорости. Для решения задачи начальное приближение строится на основе автомодельного подхода, а для обеспечения устойчивости и сходимости вычислительного процесса применяются неявные разностные схемы. Нелинейные алгебраические

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системы уравнений решаются с использованием итерационных методов и метода прогонки. В многомерном случае применяется метод переменных направлений на основе схемы Писмена–Речфорда. Результаты численных экспериментов подтверждают эффективность и надёжность предложенных алгоритмов.

Ключевые слова: математическое моделирование, нелинейные уравнения, задача Коши, численные методы, неявная схема, автомодельный подход.

Kirish

Zamonaviy fan va texnika rivojida murakkab jarayonlarni aniq va ishonchli modellashtirish muhim ahamiyat kasb etadi. Fizik, biologik, kimyoviy hamda ekologik jarayonlarning aksariyati o‘z tabiatiga ko‘ra nochiziqli bo‘lib, ularni tavsiflashda nochiziqli differensial tenglamalar va ularning sistemalaridan foydalaniladi. Bunday modellar real jarayonlarning muhim xususiyatlarini aks ettirsa-da, ularni analitik usullar yordamida yechish ko‘pincha imkonsiz yoki juda murakkab hisoblanadi. Nochiziqli differensial tenglamalar uchun qo‘yilgan boshlang‘ich va chegaraviy masalalarni o‘rganishda asosiy muammolardan biri barqaror va yaqinlashuvchi sonli algoritmlarni ishlab chiqishdir. Ayniqsa, manbali hadlarga ega bo‘lgan nochiziqli parabolik tipdagi tenglamalar sistemasi uchun Koshi masalasini yechishda boshlang‘ich yaqinlashishni to‘g‘ri tanlash hisoblash jarayonining muvaffaqiyatini belgilovchi muhim omil hisoblanadi.

Mazkur ishda ushbu muammoni hal etish uchun avtomodel yondashuv qo‘llanilib, masalaning sifat xossalarini tahlil qilish imkoniyati yaratiladi. Avtomodel o‘zgartirishlar yordamida ko‘p o‘zgaruvchili nochiziqli tenglamalar soddalashtirilib, ularni sonli usullar orqali samarali yechish mumkin bo‘ladi. Hisoblash jarayonida oshkormas ayirmali sxemalar tanlanib, ular vaqt bo‘yicha barqarorlikni ta’minlaydi, hosil bo‘ladigan nochiziqli algebraik tenglamalar sistemasi esa iteratsiya usullari va haydash algoritmi yordamida yechiladi.

Ko‘p o‘lchovli masalalar uchun o‘zgaruvchan yo‘nalishlar usuliga asoslangan Pismen–Rechford sxemasi qo‘llanilib, uning samaradorligi sonli tajribalar orqali asoslab beriladi. Olingan natijalar nochiziqli jarayonlarni matematik modellashtirish va ularni sonli yechish masalalarida amaliy ahamiyatga ega bo‘lib, turli sohalardagi modellarni tadqiq etishda qo‘llanilishi mumkin.

Asosiy qism

Zichlik va tezlik komponentlari vaqtga bog‘liq bo‘lgan manbali nochiziqli differensial tenglamalar sistemasi uchun Koshi masalasini sonli usullar yordamida yechishda eng muhim bosqichlardan biri boshlang‘ich yaqinlashishni to‘g‘ri va asosli tarzda tanlashdan iboratdir. Boshlang‘ich yaqinlashishning aniqligi va sifatli tanlanishi hisoblash jarayonining barqarorligi, iteratsion jarayonning tez

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yaqinlashuvi hamda yakuniy sonli natijalarning ishonchliligiga bevosita ta’sir ko‘rsatadi.

Mazkur ishda ushbu muammo avtomodel yondashuv asosida hal etilib, masalaning asosiy sifat xossalarini saqlab qolgan holda boshlang‘ich yaqinlashish konstruksiyasi qurib olindi. Avtomodel o‘zgartirishlar yordamida masalaning murakkabligi ma’lum darajada kamaytirilib, sonli hisoblashlarni amalga oshirish uchun qulay sharoit yaratildi. Natijada nochiziqli tenglamalar sistemasini iteratsion usullar yordamida samarali yechish imkoniyati hosil bo‘ldi.

Quyida masalaning bir o‘lchovli, ya’ni N=1N = 1N=1 holi alohida ko‘rib chiqilib, ushbu hol uchun qo‘llanilgan sonli sxemalar, iteratsiya jarayoni va hisoblash algoritmlari batafsil bayon etiladi.

$$\frac{\rho(x)\partial(u)}{\partial t} = div\left(|x|^n u^{m_1-1}|\nabla u|^{p-2}\nabla u\right) + \rho(x)\gamma(t)u^{q_1}v^{r_1} \qquad (1)$$

$$u(0,x) = u_0(x) \geq 0, x \in [0,b] \qquad (2)$$

m, k, p, p_1, q_1-musbat sonli parametrlar $u_0(x)$ komponentlar uchun mos ravishda boshlang‘ich yaqinlashish.

t va x bo‘yicha quyidagi to‘rni qurib olamiz:[108; P. 556-560]

$$\overline{w_{\tau h}} = \left\{ t_j = j\tau,\ j = 0,1,...,m,\ \tau m = T; x_i = ih,\ i = 0,1,...n,\ h = \frac{l}{n} \right\}$$

masalani taxmin qilish uchun (2.2.1).

(1)ni quyidagi ayirmali sxemaga o‘tkazamiz. Buning uchun biz oshkormas sxemadan foydalanamiz:

$$|x_i|^l \frac{y_i^{j+1} - y_i^j}{\tau} = \frac{1}{h^2}\left(|x_{i+1}|^n a_{i+1} y_i^{j+1}\left(y_{i+1}^{j+1} - y_i^{j+1}\right) - a_i |x_{i+1}|^n \left(y_i^{j+1} - y_{i-1}^{j+1}\right)\right) - \\ - |x_i|^l \gamma(t) y_i^{j+1} z_i^{j+1} \qquad (3)$$

$$i = 1,2,...,n-1; j = 0,1,....,m-1;$$
$$y_i^0 = u_0(x_i),\ i = 0,1,...,n;$$
$$y_0^j = \varphi_1(t_j),\ \ j = 1,2,...,m; \qquad (4)$$
$$y_n^j = \varphi_2(t_j),\ \ j = 1,2,....,m.$$

bilan (2.2.2) masalasining boshlang‘ich va chegara shartlari ifodalaydi. Bu yerda m, k, p, p_1, q_1 -sonli parametrlari.

Bunda quyidagi a_{i+1} va a_i - (2.2.1), (2.2.2) masalaning nochiqli hadlari. Bu holatda, a_{i+1} va a_i diffuziya jarayoni koeffitsientlari. a_{i+1} va a_i biz quyidagi ifodaga teng.

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$$a_{i+1}=\frac{1}{2}\left[\begin{array}{l}(y_i^{j+1})^{m-1}\left|\dfrac{(y_{i+1}^{j+1})^k-(y_i^{j+1})^k}{h}\right|^{p-2}+\\ +(y_{i+1}^{j+1})^{m-1}\left|\dfrac{(y_i^{j+1})^k-(y_{i-1}^{j+1})^k}{h}\right|^{p-2}\end{array}\right] \qquad (5)$$

Ko‘rinib turibdiki, (2.2.4) algebraik tenglamalar sistemasi y^{j+1} nisbatan nochiziqli. Ushbu tenglamalarni sonli yechish uchun iteratsiya usullar qo‘llaniladi.

$$|x_i|^l\frac{y_i^{j+1}-y_i^j}{\tau}=\frac{1}{h^2}\left(|x_{i+1}|^n a_{i+1}y_i^{j+1}\left(y_{i+1}^{j+1}-y_i^{j+1}\right)-a_i|x_{i+1}|^n\left(y_i^{j+1}-y_{i-1}^{j+1}\right)\right)- \\ -|x_i|^l\gamma(t)y_i^{j+1}z_i^{j+1} \qquad (6)$$

bu yerda $s=0,1,2,....$

Vaqt bo‘yicha har bir qadam uchun $\overset{s+1}{y}_i$ oldingi bosqichdagi dastlabki iteratsiya qiymatlari $\overset{0}{y}{}^{j+1}=y^j$ olinadi. Iteratsiya sxemasi bo‘yicha hisoblashda iteratsiya aniqligi beriladi va jarayon quyidagi shart bajarilgunga qadar davom etadi:

$$\max_{0\le i\le n}\left|\overset{s+1}{y}_i-\overset{s}{y}_i\right|<\varepsilon. \qquad (7)$$

Biz quyidagi belgilashlarni kiritamiz

$$y^j=y,\ y^{j+1}=\bar{y}.$$

Almashtirish bajarilganidan so‘ng (5) ayirmali sxema quyidagicha yoziladi

$$\overset{s}{A}_i\,\overset{s+1}{\bar{y}}_{i-1}-\overset{s}{C}_i\,\overset{s+1}{\bar{y}}_i+\overset{s}{B}_i\,\overset{s+1}{\bar{y}}_{i+1}=-\overset{s}{F}_i,\quad i=1,2,....n-1, \qquad (8)$$

$$y_0=\chi_1y_1+\mu_1,$$

$$y_N=\chi_2y_{N-1}+\mu_2,$$

bu yerda progonka hadlari quyidagilarga teng

$$\overset{s}{A}_i=\frac{\tau}{h^2}\overset{s}{a}_{i+1}|x_{i+1}|^m u_i^{j+1},\qquad \overset{s}{B}_i=a_{i+1}\frac{|x_{i+1}|^n}{h^2}u_i^{j+1}-\frac{|x_i|^l}{\tau}+a_i|x_{i+1}|^n+|x_i|^l\varphi(t)v_i^{j+1},$$

$$\overset{s}{C}_i=\overset{s}{A}_i+\overset{s}{B}_i+1,\qquad \overset{s}{F}_i=a_i|x_{i+1}|^n,$$

$$s=0,1,2,...,\ i=1,2,...n$$

(8) algebraik tenglamalar sistemasini sonli yechish uchun haydash usuli qo‘llaniladi. Haydash usuliga ko‘ra [108; P. 556-560]

$$\bar{y}_i=\alpha_i\bar{y}_{i+1}+\beta_i \qquad (9)$$

bu yerda α_i,β_i quyidagi formulalar bo‘yicha hisoblanadigan koeffitsientlar:

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$$\begin{cases} \alpha_{i+1} = \dfrac{B_i}{C_i - \alpha_i A_i} \\ \beta_{i+1} = \dfrac{A_i \beta_i + F_i}{C_i - \alpha_i A_i} \end{cases} \qquad (10)$$

bunda $i = 1,2,\ldots n$. Qiymatlar α_0, β_0 boshlang‘ich qiymatlar.

Yuqoridagi sonli sxemalardan foydalanib, masala sonli yechildi. Sonli tajribalar natijalari h=0.05 yetarlicha kichik oraliqlarda hisoblandi va iteratsiya sharti $\varepsilon = 10^{-3}$ aniqligida olindi. Vaqt bo‘yicha $\tau = 0.1$ qadam bilan hisoblash amalga oshirildi.

Ushbu masalani sonli yechish uchun quyidagi turdagi Pismen-Rechford sxemasi bilan o‘zgaruvchan yo‘nalishlar usuli qo‘llaniladi

$$i = 1, 2, \ldots, n-1.$$

Olingan nochiziqli tenglamalar sistemasini yechish uchun iteratsiya usuldan foydalanamiz va oshkormas sxemani quyidagicha olamiz

$$\begin{aligned} &\left(\sqrt{x_{1,i}^2 + x_{2,i}^2}\right)\frac{y_{i,j}^{l+1/2} - y_{i,j}^{l}}{0.5\cdot\tau} = \Lambda_1 y_{i,j}^{\ l+1/2} + \Lambda_2 y_{i,j}^{\ l} + |x|^{-l}\gamma(t_j)\left(y_i^{l+1/2}\right)^{q_1}\left(z_i^{l+1/2}\right)^{r_1}, \\ &\left(\sqrt{x_{1,i}^2 + x_{2,i}^2}\right)\frac{y_{i,j}^{l+1} - y_{i,j}^{l+1/2}}{0.5\cdot\tau} = \Lambda_1 y^{l+1/2} + \Lambda_2 y^{l+1} + |x|^{-l}\gamma(t_j)\left(y_i^{l+1/2}\right)^{q_1}\left(z_i^{l+1/2}\right)^{r_1} \end{aligned} \qquad (11)$$

bu yerda $i = 1, 2, \ldots, n-1$.

$$\Lambda_1 y^{l+1/2} = \frac{1}{2^{m+1}h_1^2}\left[\left(\sqrt{x_{1,i+1}^2 + x_{2,j}^2}\right)^n \left(y_{i+1,j}^l + y_{i,j}^l\right)^{m-1}\left|y_{i+1,j}^l + y_{i,j}^l\right|^{p-2}\left(y_{i+1,j}^{l+1/2} + y_{i,j}^{l+1/2}\right)\right] -$$

$$-\left(\sqrt{x_{1,i+1}^2 + x_{2,j}^2}\right)^n \left(y_{i,j}^l + y_{i-1,j}^l\right)^{m-1}\left|y_{i,j}^l + y_{i-1,j}^l\right|^{p-2}\left(y_{i,j}^{l+1/2} - y_{i-1,j}^{l+1/2}\right).$$

$$\Lambda_2 y^{l+1/2} = \frac{1}{2^{m+1}h_2^2}\left[\left(\sqrt{x_{1,i+1}^2 + x_{2,j}^2}\right)^n \left(y_{i,j+1}^l + y_{i,j}^l\right)^{m-1}\left|y_{i,j+1}^l + y_{i,j}^l\right|^{p-2}\left(y_{i,j+1}^l + y_{i,j}^l\right)\right] -$$

$$-\left(\sqrt{x_{1,i+1}^2 + x_{2,j}^2}\right)^n \left(y_{i,j}^l + y_{i,j-1}^l\right)^{m-1}\left|y_{i,j}^l + y_{i,j-1}^l\right|^{p-2}\left(y_{i,j}^l - y_{i,j-1}^l\right)$$

$$\overset{s}{A}_{i,j} = \frac{\tau}{2^{m+2}h_1^2}\left(\left|x_{i+1,j}\right|^n\right)\left(\overset{s}{\bar{y}}_{i+1,j} + \overset{s}{\bar{y}}_{i,j}\right)^{m-1}\left|\overset{s}{\bar{y}}_{i+1,j} + \overset{s}{\bar{y}}_{i,j}\right|^{p-2},$$

$$\overset{s}{B}_{i,j} = \frac{\tau}{2^{m+2}h_1^2}\left(\left|x_{i+1,j}\right|^n\right)\left(\overset{s}{\bar{y}}_{i,j} + \overset{s}{\bar{y}}_{i-1,j}\right)^{m-1}\left|\overset{s}{\bar{y}}_{i,j} + \overset{s}{\bar{y}}_{i-1,j}\right|^{p-2},$$

$$\overset{s}{C}_{i,j} = \overset{s}{A}_{i,j} + \overset{s}{B}_{i,j} + 1$$

$$\begin{cases} \overset{s}{A}_{i,j}\,\overset{s+1}{\bar{y}}_{i+1,j} - \overset{s}{C}_{i,j}\,\overset{s+1}{\bar{y}}_{i,j} + \overset{s}{B}_{i,j}\,\overset{s+1}{\bar{y}}_{i-1,j} = -\overset{s}{F}_{i,j}, \\ \bar{y} = \phi, \quad i = 0, n_1 \quad da \end{cases} \qquad (12)$$

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(12) ayirmali sxema y_i^{s+1}, z_i^{s+1} nisbatan chiziqli. Iteratsiya usuli bo‘yicha hisoblashda iteratsiyaning aniqligi belgilanadi va shartlar bajarilishi talab qilinadi

$$\max_{\substack{0\le i\le n_1\\ 0\le j\le n_2}} \left| \hat{y}_{i,j}^{s+1} - \hat{y}_{i,j}^{s} \right| < \varepsilon, \quad (13)$$

(9) da belgilanishlarni kiritish orqali pragonka hadlarini quyidagicha topiladi

$$A_{i,j}^{s} = \frac{\tau}{2^{m+2}h_1^2}\left(\bar{y}_{i+1,j}^{s} + \bar{y}_{i,j}^{s}\right)^{m-1}\left|\bar{y}_{i+1,j}^{s} + \bar{y}_{i,j}^{s}\right|^{p-2} - \nu\frac{\tau}{2h_1}$$

$$B_{i,j}^{s} = \frac{\tau}{2^{m+2}h_1^2}\left(\bar{y}_{i,j}^{s} + \bar{y}_{i-1,j}^{s}\right)^{m-1}\left|\bar{y}_{i,j}^{s} + \bar{y}_{i-1,j}^{s}\right|^{p-2}$$

$$C_{i,j}^{s} = A_{i,j}^{s} + B_{i,j}^{s} + 1$$

XULOSA

Ushbu ishda manbali nochiziqli parabolik tipdagi differensial tenglamalar sistemasi uchun Koshi masalasi sonli usullar yordamida yechildi. Boshlang‘ich yaqinlashishni avtomodel yondashuv asosida qurish hisoblash jarayonining barqarorligi va iteratsion usullarning yaqinlashuvini ta’minlashi ko‘rsatildi. Masalani yechishda oshkormas ayirmali sxemalar, iteratsiya usullari hamda haydash algoritmidan foydalanildi. Ko‘p o‘lchovli hol uchun Pismen–Rechford sxemasiga asoslangan o‘zgaruvchan yo‘nalishlar usulining samaradorligi sonli tajribalar orqali tasdiqlandi.

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