BAZI BIR UCH O‘LCHAMLI NILPOTENT ALGEBRANING DIFFERENSIALLASHI LOKAL DIFFERENSIALLASHI

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Annotatsiya: Differensiallash matematikaning fundamental tushunchalaridan biri hisoblanadi. Differensiallashlar algebra fanida ham muhim o‘rin tutadi. Differensiallashlarning turli umumlashmalari mavjud. Bular sarasiga antidifferensiallashlar, δ-differensiallashlar, ternar differensiallashlar va (α,β,γ)-differensiallashlar kiradi. Ushbu ishda uch o‘lchamli nilpotent algebralarning differensiallashi lokal differensiallashi isboti bilan ko‘rsatilgan.

Kalit so‘zlar: Differensiallash, Algebra, lokal differensiallash, chiziqli akslatirish.

Аннотатция. Дифференциация — одно из фундаментальных понятий математики. Дифференциации также играют важную роль в алгебре. Существуют различные обобщения дифференциаций. К ним относятся антидифференциации, δ-дифференциации, тернарные дифференциации и (α,β,γ)-дифференциации. В данной работе дифференциация трехмерных нильпотентных алгебр демонстрируется путем доказательства локальной дифференциации.

Ключевые слова: Дифференциация, алгебра, локальное дифференцирование, линейное отображение.

Abstract. Differentiation is one of the fundamental concepts of mathematics. Differentiations also play an important role in algebra. There are various generalizations of differentiations. These include antidifferentiations, δ-differentiations, ternary differentiations, and (α,β,γ)-differentiations. In this work, the differentiation of three-dimensional nilpotent algebras is shown by proving local differentiation.

Keywords: Differentiation, Algebra, local differentiation, linear mapping.

I. KIRISH

1-§. Masalaning qo‘yilishi

Hozirgi kunda jahonda differensiyalashlar nazariyasi bilan bir qatorda, operator algebralarida lokal va 2-lokal differensiyalashlar nazariyasi ham muhim hisoblanadi. So‘ngi 20 yil davomida fon Neyman algebralari C^* - algebralari va JB^*

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- uchliklarda lokal va 2- lokal differensiallashlar nazariyasini o‘rganish bo‘yicha samarali natijalarga erishildi. Lokal differensiallashlarni o‘rganish 1990-yilda R.B.Kadison va D.R.Larson hamda A.S.Sururlar tomonidan boshlangan.

Ixtiyoriy uch o‘lchamli nilpotent algebra bazislari $\{e_1, e_2, e_3\}$ bo‘lgan quyidagi o‘zaro izomorf bo‘lmagan algebralarning biriga izomorf bo‘ladi:

N_1: $e_1^2=e_2$,$e_2^2 = e_3$ N_2: $e_1^2 = e_2$, $e_2e_1 = e_3$, $e_2^2 = e_3$

N_3: $e_1^2 = e_2$, $e_2e_1 = e_3$ $N_4(\alpha)$: $e_1^2 = e_2$, $e_1e_2 = e_3$, $e_2e_1 = \alpha e_3$

N_5: $e_1^2 = e_2$ N_6: $e_1^2 = e_3$, $e_2^2 = e_3$

N_7: $e_1e_2 = e_3$, $e_2e_1 = -e_3$ $N_8(\alpha)$: $e_1^2 = \alpha e_3$, $e_2e_1 = e_3$, $e_2^2 = e_3$

II.ADABIYOTLAR TAHLILI

Differensiallashning ko‘plab umumlashmalari mavjud. Eng asosiy umumlashmasi bu lokal va 2-lokal umumlashmalari hisoblanadi. So‘ngi yillarda ko‘plab olimlar lokal va 2- lokal differensiallashlarga oid ko‘plab maqolalar chop etishdi[1- 3]

R haqiqiy sonlar maydoni ustida aniqlangan $\{e_1, e_2, e_3\}$ bazisli nilpotent algebra N_5 berilgan. Bu algebrada ko‘paytirish jadvali quyidagicha aniqlanadi:

$$e_1^2 = e_2$$

N_5 algebrada ihtiyoriy x vektorni

$$x = x_1e_1 + x_2e_2 + x_3e_3$$

ko‘rinishda yozishmumkin. $d : N_5 \to N_5$akslantirishni qaraylik.

1-Ta’rif. Agar N algebaradan olingan ixtiyoriy x, y elementlar uchun

$$d(x \cdot y) = d(x) \cdot y + x \cdot d(y)$$

Tenglik bajarilsa d akslantirish qaralayotgan N algebrada differensiallash deb ataladi.

1-tarifga ko‘ra $d(x) = Ax$ chiziqli akslantirishni qaraylik, bu yerda A – 3×3 o‘lchamli kvadrat matritsa. Ravshanki, agar N_5 algebaradan olingan ixtiyoriy x, y elementlar uchun

$$A(xy) = (Ax)y + x(Ay)$$

Tenglik bajarilsa $d(x) = Ax$ akslantirish qaralayotgan N_5 algebrada differensiallashdan iborat bo‘ladi.

1-Teorema. $d(x) = Ax$ akslantirish N_5 algebrada differensiallashdan iborat bo‘lishi uchun A matritsa

$$A = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & a_{2,3} \\ a_{3,1} & 0 & a_{3,3} \end{pmatrix} \qquad (6)$$

ko‘rinishda bo‘lishi zarur va yetarli.

Isbot. Zarurligi. $d(x) = Ax$ akslantirish N_5 algebrada differensiallashdan iborat bo‘sin. U holda

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$$A=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

Matritsa ixtiyoriy x , y vektorlar uchun (1) tenglikni qanoatlantirishi kerak.

Xususan $e_1e_1=e_2$ tenglikni xisobga olsak $A(e_1e_1)=A(e_2)$ yoki

$$(Ae_1)e_1+e_1(Ae_1)=A(e_2)$$

Tenglik bajariladi. Xisoblashlar bajaraylik

$$Ae_1=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} a_{11} \\ a_{21} \\ a_{3,1} \end{pmatrix}=a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3$$

$$Ae_2=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}=\begin{pmatrix} a_{1,2} \\ a_{2,2} \\ a_{3,2} \end{pmatrix}=a_{1,2}e_1+a_{2,2}e_2+a_{3,2}e_3$$

$$(Ae_1)e_1=(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3)e_1=a_{1,1}e_2$$

$$e_1(Ae_1)=e_1(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3)=a_{1,1}e_2$$

$$a_{1,1}e_2+a_{1,1}e_2=a_{1,2}e_1+a_{2,2}e_2+a_{3,2}e_3$$

bundan

$$a_{2,2}=2a_{1,1}\quad,\quad a_{1,2}=0\quad,\quad a_{3,2}=0$$

$$Ae_2=2a_{1,1}e_2$$

tengliklarni aniqlaymiz.

Endi $e_1e_2=0$ tenglikni xisobga olsak $A(e_1e_2)=A(0)$ yoki

$$(Ae_1)e_2+e_1(Ae_2)=A(0)$$

Tenglik bajariladi.

40

$$(Ae_1)e_2=(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3)e_2=0$$

$$e_1(Ae_2)=e_12a_{1,1}e_2=0$$

$$0=0$$

bo‘ladi.

Endi $e_1e_3=0$ tenglikni xisobga olsak $A(e_1e_3)=A(0)$ yoki

$$(Ae_1)e_3+e_1(Ae_3)=A(0)$$

tenglik bajariladi.

$$(Ae_1)e_3=(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3)e_3=0$$

$$e_1(Ae_3)=e_1(a_{1,3}e_1+a_{2,3}e_2+a_{3,3}e_3)=a_{1,3}e_2$$

$$0+a_{1,3}e_2=0$$

bundan

$$a_{1,3}=0\quad,\quad Ae_3=a_{2,3}e_2+a_{3,3}e_3$$

tengliklarni aniqlaymiz.

Endi $e_2e_1=0$ tenglikni xisobga olsak $A(e_2e_1)=A(0)$ yoki

$$(Ae_2)e_1+e_2(Ae_1)=A(0)$$

ISSN 3030-3907

tenglik bajariladi.

$$(Ae_2)e_1 = 2a_{1,1}e_2e_1 = 0$$
$$e_2(Ae_1) = e_2(a_{1,1}e_1 + a_{2,1}e_2 + a_{3,1}e_3) = 0$$
$$0 = 0$$

bo‘ladi.

Endi $e_2e_2 = 0$ tenglikni xisobga olsak $A(e_2e_2) = A(0)$ yoki

$$(Ae_2)e_2 + e_2(Ae_2) = A(0)$$

tenglik bajariladi.

$$(Ae_2)e_2 = 2a_{1,1}e_2e_2 = 0$$
$$e_2(Ae_2) = e_2 2a_{1,1}e_2 = 0$$
$$0 = 0$$

41

bo‘ladi.

Endi $e_2e_3 = 0$ tenglikni xisobga olsak $A(e_2e_3) = A(0)$ yoki

$$(Ae_2)e_3 + e_2(Ae_3) = A(0)$$

tenglik bajariladi.

$$(Ae_2)e_3 = 2a_{1,1}e_2e_3 = 0$$
$$e_2(Ae_3) = e_2(a_{2,3}e_2 + a_{3,3}e_3) = 0$$
$$0 = 0$$

bo‘ladi.

Endi $e_3e_1 = 0$ tenglikni xisobga olsak $A(e_3e_1) = A(0)$ yoki

$$(Ae_3)e_1 + e_3(Ae_1) = A(0)$$

tenglik bajariladi.

$$(Ae_3)e_1 = (a_{2,3}e_2 + a_{3,3}e_3)e_1 = 0$$
$$e_3(Ae_1) = e_3(a_{1,1}e_1 + a_{2,1}e_2 + a_{3,1}e_3) = 0$$
$$0 = 0$$

bo‘ladi.

Endi $e_3e_2 = 0$ tenglikni xisobga olsak $A(e_3e_2) = A(0)$ yoki

$$(Ae_3)e_2 + e_3(Ae_2) = A(0)$$

tenglik bajariladi

$$(Ae_3)e_2 = (a_{2,3}e_2 + a_{3,3}e_3)e_2 = 0$$
$$e_3(Ae_2) = e_3 2a_{1,1}e_2 = 0$$
$$0 = 0$$

bo‘ladi.

Endi $e_3e_3 = 0$ tenglikni xisobga olsak $A(e_3e_3) = A(0)$ yoki

$$(Ae_3)e_3 + e_3(Ae_3) = A(0)$$

tenglik bajariladi

$$(Ae_3)e_3 = (a_{2,3}e_2 + a_{3,3}e_3)e_3 = 0 \quad , \quad e_3(Ae_3) = e_3(a_{2,3}e_2 + a_{3,3}e_3) = 0$$

ISSN 3030-3907

$$0 = 0$$

bo‘ladi.

Yetarliligi. Endi A matritsa (1) ko‘rinishda bo‘lsa $d(x) = Ax$ chiziqli akslantirish differensiallashdan iboratligini ko‘rsataylik.

$$xy = (x_1e_1 + x_2e_2 + x_3e_3)(y_1e_1 + y_2e_2 + y_3e_3) =$$
$$= x_1y_1e_2$$

tenglikka ko‘ra

$$A(xy) = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & a_{2,3} \\ a_{3,1} & 0 & a_{3,3} \end{pmatrix} \begin{pmatrix} 0 \\ x_1y_1 \\ 0 \end{pmatrix} =$$
$$2a_{1,1}x_1y_1e_2$$

boshqa tamondan

$$Ax = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & a_{2,3} \\ a_{3,1} & 0 & a_{3,3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = a_{1,1}x_1e_1 + (a_{2,1}x_1 + 2a_{1,1}x_2 + a_{2,3}x_3)e_2 +$$
$$(a_{3,1}x_1 + a_{3,3}x_3)e_3$$
$$(Ax)y = a_{1,1}x_1e_1 + (a_{2,1}x_1 + 2a_{1,1}x_2 + a_{2,3}x_3)e_2 +$$
$$(a_{3,1}x_1 + a_{3,3}x_3)e_3)(y_1e_1 + y_2e_2 + y_3e_3) = a_{1,1}x_1y_1e_2$$

$$Ay = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & a_{2,3} \\ a_{3,1} & 0 & a_{3,3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = a_{1,1}y_1e_1 + (a_{2,1}y_1 + 2a_{1,1}y_2 + a_{2,3}y_3)e_2 +$$
$$(a_{3,1}y_1 + a_{3,3}y_3)e_3$$
$$x(Ay) = (x_1e_1 + x_2e_2 + x_3e_3)(a_{1,1}y_1e_1 + (a_{2,1}y_1 + 2a_{1,1}y_2 + a_{2,3}y_3)e_2 +$$
$$(a_{3,1}y_1 + a_{3,3}y_3)e_3 = a_{1,1}x_1y_1e_2$$
$$(Ax)y + x(Ay) = a_{1,1}x_1y_1e_2 + a_{1,1}x_1y_1e_2 = 2a_{1,1}x_1y_1e_2$$

Yuqoridagi xisoblashlar (1) tenglik to‘g‘riligini ko‘rsatadi.

Teorema isbotlandi.

2-Teorema. Agar Δ matritsasi pastki uchburchak shaklda bo‘lsa, N_5 dagi Δ chiziqli akslantirish lokal differensiallash bo`ladi.

Isboti. Δ- N_5 bo‘yicha ixtiyoriy lokal differensiallashi bo‘lsin va Δ matritsasi

$$A = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

bo‘lsin. Barcha $x = x_1e_1 + x_2e_2 + x_3e_3$ uchun ta’rifga ko‘ra N_5 ustida Adifferensiallash mavjudki

$$\Delta(x) = A_x(x)$$

tenglik o`rinli.

2-teoremaga ko`ra $A_x$ differensiallash quyidagi ko`rinishga ega bo`lsin

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$$A_x = \begin{pmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & 2a_{1,1} & a_{2,3} \\ a_{3,1} & 0 & a_{3,3} \end{pmatrix} .$$

$x = e_1, x = e_2, x = e_3$larni tanlab va $\Delta(x) = A_x(x)$ tenglikdan foydalanib, ya`ni $A\bar{x} = A_x\bar{x}$ ga ko`ra $b_{12} = b_{13} = b_{32} = 0$ natijaga ega bo`lamiz. Bu yerda $\bar{x} - x$ ga mos vektor. Bundan A matritsa

$$A = \begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & b_{23} \\ b_{31} & 0 & b_{33} \end{pmatrix}$$

ko`rinishga keladi.

Yana $\nabla(x) = A_x(x)$ dan foydalanib, ya'ni $A\bar{x} = A_x\bar{x}$ ga ko`ra

$$\begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & b_{23} \\ b_{31} & 0 & b_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha_x & 0 & 0 \\ \beta_x & 2\alpha_x & \delta_x \\ \gamma_x & 0 & \varepsilon_x \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Tenglamalar sistemasiga ega bo`lamiz.

$$\begin{cases} \alpha_x x_1 = b_{1,1}x_1 \\ \beta_x x_1 + 2\alpha_x x_2 + \delta_x x_3 = b_{2,1}\cdot x_1 + b_{2,2}\cdot x_2 + b_{2,3}\cdot x_3 \\ \gamma_x x_1 + \varepsilon_x x_3 = b_{3,1}x_1 + b_{3,3}x_3 \end{cases}$$

Agar $x_1 \neq 0$bo‘lsa

$$\alpha_x = b_{1,1}$$

$$\beta_x = b_{2,1} + (b_{2,2} - 2\alpha_x)\frac{x_2}{x_1} + (b_{3,2} - \delta_x)\frac{x_3}{x_1}$$

$$\gamma_x = b_{3,1} + (b_{3,3} - \varepsilon_x)\frac{x_3}{x_1}$$

Agar $x_1 = 0,\ x_2 \neq 0$ bo‘lsa,

$$2\alpha_x = b_{2,2} + (b_{3,2} - \delta_x)\frac{x_3}{x_2}$$

$$\varepsilon_x = b_{3,3}$$

Agar $x_1 = x_2 = 0,\ x_3 \neq 0$ bo‘lsa,

$$\delta_x = b_{3,2}.$$

Teorema isbotlandi.

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