PROPAGATION OF LINEAR WAVES IN MULTILAYERED STRUCTURAL - INHOMOGENEOUS CYLINDRICAL SHELLS

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Annotation. In this paper, axisymmetric viscoelastic waves in an extended cylindrical shell of circular cross section are considered. The study of axisymmetric viscoelastic waves in multilayer cylindrical systems with the maximum possible use of analytical methods is relevant for the development of a new generation of sounding techniques for wells. The aim of the work is to study the propagation of axisymmetric viscoelastic waves in extended multilayer cylindrical structures, to determine the volumetric structure variant from the responses of one-way sounding using a priori information about the possible structure of the system. The paper uses classical methods of mathematical physics, applied to solving boundary value problems in a cylindrical coordinate system, the spatial Fourier transform, the method of complex amplitudes for the variable components of the displacement vector and the stress tensor. The low-frequency resonances of a cylindrical shell are investigated. The physical laws of the formation of the pitch of its fundamental tone are determined sounding. The impossibility of propagation along the casing of wells is proved low-frequency surface waves, as well as SH-polarization waves at axisymmetric excitation. The thin-walled cylindrical shell in the low-frequency region has two fundamental resonance associated with the resonance of the shell as a whole and resonance of each individual section as a ring. Beats of these frequencies the basic tone of the sound of an extended shell of this type is determined. At frequencies below the cutoff of higher modes, all the features of the dispersion natural waves in the wall of a circular cylinder shell are described approximate algebraic equation of the third degree. One the real root of this equation corresponds to the zero mode antisymmetric Lamb wave, and one of the other two complex conjugate, describes the zero mode of the symmetric wave.

Keywords: cylindrical shell, resonance, viscoelastic wave, beat, zero mode, axisymmetric excitation.

Annotatsiya. Ushbu hujjatda dumaloq kesmaning kengaytirilgan silindrsimon qobig'idagi eksimmetrik viskoelastik to'lqinlar ko'rib chiqiladi.zoh. Ushbu hujjatda dumaloq kesmaning kengaytirilgan silindrsimon qobig'idagi eksimmetrik viskoelastik to'lqinlar ko'rib chiqiladi. Analitik usullardan maksimal darajada foydalangan holda ko'p qatlamli silindrsimon tizimlarda aksisimmetrik viskoelastik to'lqinlarni o'rganish quduqlar uchun yangi avlod tovush texnikasini ishlab chiqish uchun dolzarbdir. Ishning maqsadi kengaytirilgan ko'p qatlamli silindrsimon tuzilmalarda aksisimmetrik viskoelastik to'lqinlarning tarqalishini

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o'rganish, tizimning mumkin bo'lgan tuzilishi haqida bir tomonlama tovushlarning javoblaridan volumetrik tuzilish variantini aniqlashdir.shning maqsadi kengaytirilgan ko'p qatlamli silindrsimon tuzilmalarda aksisimmetrik viskoelastik to'lqinlarning tarqalishini o'rganish, tizimning mumkin bo'lgan tuzilishi haqida bir tomonlama tovushlarning javoblaridan volumetrik tuzilish variantini aniqlashdir. Qog'oz matematik fizikaning klassik usullaridan foydalanadi, bu chegara muammolarini hal qilishda qo'llaniladi.silindrsimon koordinata tizimi, fazoviy Furye konvertatsiyasi, siljish vektorining o'zgaruvchan komponentlari va stress tensori uchun murakkab amplitudalar usuli. Silindrsimon qobiqning past chastotali rezonanslari o'rganilmoqda.ilindilindrsimon qobiqning past chastotali rezonanslari o'rganilmoqda. Uning asosiy ohangini shakllantirishning fizik qonunlari aniqlanadi. Quduqlar korpusi bo'ylab tarqaliilindrsimon qobiqning past chastotali rezonanslari o'rganilmoqda. Uning asosiy ohangini shakllantirishning fizik qonunlari aniqlanadi. Quduqlar korpusi bo'ylab tarqalishning mumkin emasligi isbotlangan past chastotali sirt to'lqinlari, shuningdek sh-polarizatsiya to'lqinlari eksimmetrik qo'zg'alishda. Past chastotali mintaqadagi yupqa devorli silindrsimon qobiq ikkita asosiy rezonansga ega bo'lib, qobiqning rezonansi butun va har bir alohida qismning rezonansi bilan bog'liq. Ushbu chastotalarning urishlari ushbu turdagi kengaytirilgan qobiq tovushining asosiy ohangi aniqlanadi.shbu chastotalarning urishlari ushbu turdagi kengaytirilgan qobiq tovushining asosiy ohangi aniqlanadi. Yuqori rejimlarning kesilishidan past chastotalarda, dispersiyaning barcha xususiyatlari tabishbu chastotalarning urishlari ushbu turdagi kengaytirilgan qobiq tovushining asosiy ohangi aniqlanadi.

Kalit so'zlar: silindrsimon qobiq, rezonans, viskoelastik to'lqin, urish, nol rejim, aksisimmetrik qo'zg'alish.

Аннотация. В данной работе рассматриваются осесимметричные вязкоупругие волны в протяженной цилиндрической оболочке круглого поперечного сечения. Изучение осесимметричных вязкоупругих волн в многослойных цилиндрических системах с максимально возможным использованием аналитических методов актуально для разработки нового поколения методов зондирования скважин. Целью работы является изучение распространения осесимметричных вязкоупругих волн в протяженных многослойных цилиндрических конструкциях, определение варианта объемной структуры по откликам одностороннего зондирования с использованием априорной информации о возможной структуре системы. В работе использованы классические методы математической физики, применяемые для решения краевых задач в цилиндрической системе координат, пространственное преобразование Фурье, метод комплексных

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амплитуд для переменных составляющих вектора перемещений и тензора напряжений. Исследованы низкочастотные резонансы цилиндрической оболочки. Определены физические закономерности формирования высоты ее основного тона. Доказана невозможность распространения вдоль обсадной колонны скважин низкочастотных поверхностных волн, а также волн SH-поляризации при осесимметричном возбуждении. Тонкостенная цилиндрическая оболочка в низкочастотной области имеет два основных резонанса, связанных с резонансом оболочки в целом и резонансом каждой отдельной секции в виде кольца. Биениями этих частот определяется основной тон звучания протяженной оболочки данного типа. На частотах, которые ниже границы отсечения высших мод, все особенности рассеяния собственных волн в стенке круглой цилиндрической оболочки описываются приближенным алгебраическим уравнением третьей степени. Один действительный корень этого уравнения соответствует нулевой моде антисимметричной волны Лэмба, а один из двух других комплексно сопряженных описывает нулевую моду симметричной волны.

Ключевые слова: цилиндрическая оболочка, резонанс, вязкоупругая волна, биение, нулевая мода, осесимметричное возбуждение.

1. Introduction

In this paper, wave processes in an extended (end faces considered remote) circular cylindrical shell are studied, an example of which (in practice) can be a well casing or any sufficiently long straight pipe of constant cross section. A similar object is the subject of consideration at once of two relatively independently developing directions of the theory of elasticity. On the one hand, traditionally, a cylindrical shell is studied in the theory of shells [1–5], on the other hand, it is of great interest for the theory of elastic waveguides [6–9]. Indeed, in most applications, the length of the shell is small, and the influence of the edges is significant, and the vibrations, even if they occur, are low-frequency. As a result, the wavelength turns out to be longer or comparable with the largest shell size, and there is no need to talk about the wave process in it. On the contrary, the directed propagation of elastic waves is most often considered in bodies whose geometry does not allow them to be attributed to shells.. The sewerage of waves along smooth surfaces (Rayleigh waves) and regular (edges and troughs) or stochastic surface inhomogeneities at the interface is studied in [10], as well as in the thickness of continuous continuous bodies of finite linear volume (rods of various shapes) [11] . However, when it comes to a subject equally related to both sections, as happens with pipes, difficulties arise due to the separation by specialization, which sometimes makes it difficult to look at some phenomena with the original physical community. Significant successes of the cladding theory and the universality of the

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approaches used in it, apparently, have led to the fact that the influence of its "quasistatic" views has also affected the study of elastic waveguides. In any case, pipes are treated primarily as shells, and they use the results of the theory of shells, even if they are very long, in those problems where the wave processes in them are most important. As a result, even dispersion dependences can only be found numerically, as evidenced by many works [12, 13, 14].

It should be noted that even if approximations (Kirchhoff, Tymoshenko, etc.) are used in the theory of shells, these solutions, strictly speaking, are not exact. In addition, in some works, elastic waves are studied in pipes immediately upon their contact with filling the space around them. This can be either water [15] or a solid medium, for example, rock in seismic problems considered in [16, 17]. In both cases, the properties of the shell itself are not fully revealed, and due to the initial complexity of the task, the search for analytical solutions where they could clearly stand out from the properties of the composite structure also becomes almost impossible. Also, in recent years, there has been a tendency to complicate the problem by additional consideration of the so-called helical, non-axisymmetric waves, which are devoted to a number of works, for example [18, 19]. It is important to note that, as was convincingly shown in [13], in the equilibrium equations of the cylindrical shell of the most general form, the azimuthal and radial variables are separated, and thus twisting waves around the axis and axisymmetric waves, forming together a complete set of possible waves, can be considered strictly separately. Since the present study is devoted only to axisymmetric structures and excitations, twisting waves will not be considered further, and it is significant that, based on the foregoing, this does not limit the generality of considering types of axisymmetric waves. For this work, the initial wave approach to solving the problem of the propagation of vibrations in the wall of an extended circular cylindrical shell is specific: first of all, taking into account the selected propagation directions (axis), waves are searched for in an infinite isotropic elastic medium. Moreover, the behavior of the material at each point (the fulfillment of the continuity condition [20], etc.) corresponds to the assumptions that underlie the most general laws of the theory of elasticity: Hooke's law, continuity equations, and equations of motion for a solid [20], of which and the Lame equation is derived. It is significant that in the theory of elasticity and in the theory of shells, the vibrations are considered small. Thus, the relationship between displacements and stresses is linear, and the boundaries are fixed in the sense of constant time coordinates of the imposition of conditions on them (that is, despite elastic waves, the front surfaces are considered non-deformable). From the equivalence of the initial postulates it follows that the results of the solution by the methods of elasticity theory are at least equivalent to the results of the solution by the methods

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of shell theory. Of course, the wave approach is advisable only in the study of extended shells, but for a problem with a homogeneous waveguide of a relatively simple form, it allows one to analytically obtain the main dependences, since it does not need additional approximations. Obtaining the result in an analytical form became possible due to the fact that the topological features of the cross-sectional shape can be taken into account, nevertheless, considering the wall to be everywhere locally flat: when boundary conditions are imposed, the radius of curvature of the shell is assumed to be large compared to the wall thickness. It turns out that the cylindrical functions in the approximate dispersion equations are reduced for each degree of a small parameter, but keeping an increasing number of members of the series in the expansion of the boundary conditions refines the correction for cylindricity. In the limit, when the number of members of the series tends to infinity, these cylindrical functions will undoubtedly be obtained in the same way as they are determined by searching for solutions to the corresponding equations in mathematical physics.

As each new member of the series adds new roots todispersion equation, a convenient way to classify waves according to the relative magnitude of their contribution to the overall picture is obtained already at the stage of constructing the dispersion curve (or taking into account the specifics of excitation). For example, it will be shown that a symmetric wave in a thin-walled cylindrical shell is next in order of smallness after an antisymmetric one. A similar separation takes place in the theory of shells, but there it does not lead to analytical expressions, but here the dispersion equation is algebraic, and, moreover, its roots can be explicitly expressed in elementary functions in at least the first and second approximations. Another advantage of the waveguide approach is the possibility of decomposing the complete wave process into the Fourier integral along the coordinate of the axis, that is, on a continuum of conical waves. Here, another simplifying geometric factor is taken into account - the invariance of the shape of the section along the axis.

The theory of shells is designed for their potentially more complex configurations and is applicable to thin bodies of any curvature that may have edges (ends), cutouts, etc., but such complications are not investigated in this paper. As curvilinear coordinates on the middle (or front) surface in the considered simple case of a circular cylinder, ordinary three-dimensional cylindrical coordinates are completely suitable, for which it is known how to solve the corresponding equation - cylindrical functions, and there is a ready-made integral transformation - the Bessel transform, which allows instead of a system of differential equations solve a simple system of algebraic equations (Fourier-Bessel transform method).

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Despite the huge number of numerical results, obtaining analytical dependences seems to be very important. All obtained approximate analytical dependences are compared with the numerical solution of exact equations for the same parameters.

2. Statement of the problem and methods for solving the propagation of waves in a viscoelastic layered cylindrical shell

In a cylindrical coordinate system $\{r, \theta, z\}$ the problem of the propagation of natural waves in an isotropic viscoelastic shell consisting of N $(k = 1, N)$ cylindrical shells (Figure 1).

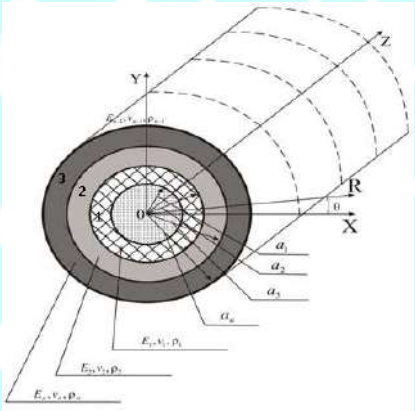


Figure 1. The design scheme of the layered shell

The linear equation of motion of a mechanical system, in vector form in the absence of volume forces, takes the form:

$$\tilde{\mu}_\kappa \nabla^2 \vec{u} + (\tilde{\lambda}_\kappa + \tilde{\mu}_\kappa)\, graddiv\ \vec{u} = \rho_\kappa \frac{\partial^2 \vec{u}}{\partial t^2}\ , (\kappa = 1, 2, 3 \ldots .N.) \qquad (1)$$

Where

$$\tilde{\lambda}_\kappa \phi(t) = \frac{v_\kappa}{(1+v_\kappa)(1-2v_\kappa)} \tilde{E}_\kappa \phi(t);$$

$$\tilde{\mu}_\kappa \phi(t) = \frac{v_\kappa}{2(1+v_\kappa)} \tilde{E}_\kappa \phi(t),$$

$$\tilde{E}_\kappa \phi(t) = E_{\kappa 0} \left[\phi(t) - \int_0^t R_{E\kappa}(t-\tau)\phi(t)\, d\tau \right], \qquad (2)$$

$\phi(t)$– arbitrary function of time; t is time; $R_{Ek}(t-\tau)$– core relaxation; $E_{\kappa 0}$- instant elastic moduli (κ=1……N); $\vec{u}$ –Vector of displacements; ρ_k- medium density, k- ordinal number of layers.

It is assumed that the integral terms are sufficiently small, then we can imagine $\varphi(t) = \psi(t) e^{-i\omega_R t}$. Here $\psi(t)$- slowly changing function. In this case, relations (2) can be replaced by the following relations (freezing method [20])

$$\bar{E}_\kappa \phi = E_{k0} \left[1 - \Gamma_\kappa^C(\omega_R) - i\Gamma_\kappa^S(\omega_R) \right] \phi,$$

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Where

$$\Gamma_\kappa^{\,C}(\omega_R)=\int_0^\infty R_{Ek}(\tau)\cos\omega_R\tau\,d\tau,\ \Gamma_\kappa^{\,S}(\omega_R)=\int_0^\infty R_{Ek}(\tau)\sin\omega_R\tau\,d\tau\ .$$

Between the layers, the conditions of hard (or sliding) contact are set [21]:

$$r=R_\kappa;\quad \sigma_{rr\kappa}=\sigma_{rr(\kappa+1)};\quad \sigma_{r\theta\kappa}=\sigma_{r\theta z(\kappa+1)};\ \sigma_{rz\kappa}=\sigma_{rz(\kappa+1)};$$
$$u_\kappa=u_{\kappa+1};\quad \vartheta_\kappa=\vartheta_\kappa;\quad w_\kappa=w_{\kappa+1} \qquad (3)$$

If the outer surface of the multilayer cylinder is stress free, then

$$r=R_{\kappa+1};\quad \sigma_{rr(\kappa+1)}=0;\quad \sigma_{r\theta(\kappa+1)}=0;\quad \sigma_{rz(\kappa+1)}=0\,.$$

If the displacement vector is represented in the form of potential and solenoidal parts

$$\vec{u}=grad\ \phi_\kappa+rot\,\vec{\psi}_\kappa, div\vec{\psi}_\kappa=0,$$

then the wave equation has the form

$$\Delta\phi_k-\frac{1}{c_{pj}^2}\frac{\partial^2\phi_k}{\partial t^2}=0;$$

$$\Delta\psi_{zk}-\frac{1}{c_{sj}^2}\frac{\partial^2\psi_{zk}}{\partial t^2}=0;$$

$$\Delta\psi_{rk}-\frac{\psi_{rk}}{r^2}+\frac{2}{r^2}\frac{\partial\psi_{\theta k}}{\partial\theta}-\frac{1}{c_{sk}^2}\frac{\partial^2\psi_{rk}}{\partial t^2}=0;$$

$$\Delta\psi_{\theta k}-\frac{\psi_{\theta k}}{r^2}+\frac{2}{r^2}\frac{\partial\psi_{rk}}{\partial\theta}-\frac{1}{c_{sk}^2}\frac{\partial^2\psi_{\theta k}}{\partial t^2}=0; \qquad (4)$$

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2};$$

$$u_{rk}=\frac{\partial\phi_k}{\partial r}+\frac{1}{r}\frac{\partial\psi_{zk}}{\partial\theta}-\frac{\partial\psi_{\theta k}}{\partial z};$$

$$u_{\theta k}=\frac{1}{r}\frac{\phi_k}{\partial\theta}+\frac{\partial\psi_{zk}}{\partial z}-\frac{\partial\psi_{\theta k}}{\partial r};$$

$$u_{zk}=\frac{\partial\phi_k}{\partial r}+\frac{\partial\psi_{\theta k}}{\partial r}+\frac{\partial\psi_{zk}}{\partial r}-\frac{1}{r}\frac{\partial\psi_{rk}}{\partial\theta},\ k=1,2,\ldots N.$$

Where $\phi(r,\theta,z,t)$ – longitudinal wave potential; $\vec{\psi}\ (\psi_r,\psi_\theta,\psi_z)$ - shear wave potential, $\Gamma_\kappa=1-\Gamma^C(\omega_R)-i\Gamma^S(\omega)$; $c_{pk}^2=((\lambda_k+2\mu_k)/\rho_k)\Gamma_k$; $c_{sk}^2=(\mu_k/\rho_k)\Gamma_k$. When constructing representations for components of the displacement vector in a multilayer cylinder

$$0<r_1\le a_1;\quad a_1\le r_2\le a_2;\ldots a_{k-1}\le r_k\le a_k;\quad a_{n-1}\le r_n\le\infty\quad (k=1,2,\ldots,N)\,|z_1|<\infty$$

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we proceed from equation (4) for the scalar $\phi(r,\theta,z,t)$ and stock $\vec{\psi}(r,\theta,z,t)$ potentials.

The geometry of the object and the natural assumption about the nature of the wave motion along the OZ axis make it possible to substantially predict the shape of the desired scalar and vector functions. They should represent waves traveling along the OZ axis. We seek solutions of equations (4) in the form:

$$\left.\begin{aligned}
\phi_k(r,\theta,z,t)&=\sum_{n=0}^{\infty}\phi_n(\alpha_k r)\begin{Bmatrix}\cos n\theta\\ -\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{rk}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\psi_{nr}(\beta_k r)\begin{Bmatrix}\sin n\theta\\ -\cos n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{\theta k}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\psi_{n\theta}(\beta_k r)\begin{Bmatrix}\cos n\theta\\ -\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{zk}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\psi_{nz}(\beta_k r)\begin{Bmatrix}\sin n\theta\\ \cos n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};
\end{aligned}\right\} \qquad (5)$$

Where n - integer; γ_{pk} - constant wave propagation; $\omega=2\pi f$ - natural frequency; $r=\frac{r_1}{a_0}, z=\frac{z_1}{a_0}$. At infinity $(r\to\infty)$ Sommerfeld conditions are set for each component.

Substituting (5) in (4), we obtain the following ordinary differential equations:

$$\left.\begin{aligned}
&\frac{d^2\phi_k}{dr^2}+\frac{1}{r}\frac{d\phi_k}{dr}+\left(\alpha_k^2-\frac{n^2}{r^2}\right)\phi_k=0;\\
&\frac{d^2\psi_{zk}}{dr^2}+\frac{1}{r}\frac{d\psi_{zk}}{dr}+\left(\beta_k^2-\frac{n^2}{r^2}\right)\psi_{zk}=0;\\
&\frac{d^2\psi_{\theta k}}{dr^2}+\frac{1}{r}\frac{d\psi_{\theta k}}{dr}+\frac{1}{r^2}\left(-n^2\psi_{\theta k}+2n\psi_{\theta k}-\psi_{\theta k}\right)\beta^2\psi_{\theta k}=0;\\
&\frac{d^2\psi_{rk}}{dr^2}+\frac{1}{r}\frac{d\psi_{rk}}{dr}+\frac{1}{r^2}\left(-n^2\psi_{rk}+2n\psi_{\theta k}-\psi_{rk}\right)\beta^2\psi_{rk}=0;
\end{aligned}\right\} \qquad (6)$$

Where $\alpha_k^2=\frac{\Omega_k^2}{\gamma_k^2}-\gamma_p^2;\quad \beta_k^2=\Omega_k^2-\gamma_p^2;\quad \Omega_k=\frac{\omega\alpha_k}{c_{sk}};\quad \gamma_k^2=\frac{2(1-\nu_k)}{1-2\nu_k}.$

The first two equations in (6) have the following solutions for the first k = 1 (solid) and external (k = N) cylinders:

$$\phi_k(r)=\begin{cases}F_{nk}J_n(\alpha_k r), & k=1;\\ F'_{nk}H_n^{(1)}(\alpha_k r), & k=N;\end{cases}\quad \psi_z(r)=\begin{cases}M_{1k}J_n(\beta_k r), & k=1;\\ M'_{1k}H_n^{(1)}(\beta_k r), & k=N,\end{cases} \qquad (7)$$

Where $J_n(\alpha_k r)$ - n-th order Bessel function, $H_n^{(1)}(\beta_k r)$ - Hankel function of the first kind of the nth order.

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The solution of the other two equations in (6) is also expressed in terms of the Bessel and Hankel functions:

$$\psi_{rk}(r)=\begin{cases}L_{1nk}J_{n-1}(\beta_k r)+L_{2nk}J_{n+1}(\beta_k r), & k=1;\\ L^{/}_{1nk}H^{(1)}_{n-1}(\beta_k r)+L^{/}_{2nk}H^{(2)}_{n+1}(\beta_k r), & k=N;\end{cases} \quad (8)$$

Solutions (7, a), (7, b) and (8) of the system of differential equations (6) will contain 6k-2 arbitrary constants. Taking the opportunity to act largely arbitrarily when choosing the values of one of the constants, we assume further, $L_{1k}=L^{/}_{1k}=0$, т.е. $\psi_{rk}=\psi_{\theta k}$.

The displacements of the cylinder are expressed in terms of the n-order Bessel and Neumann functions of the complex argument:

$$u_{r\kappa}=\sum_{n=0}^{\infty}\left\{\gamma_\kappa\left[A_{1\kappa n}J'_n(\gamma_{1\kappa}r)+A_{2\kappa n}Y'_n(\gamma_{1\kappa}r)\right]+\frac{n}{r}\left[A_{3\kappa n}J_n(\gamma_{2\kappa}r)+A_{4\kappa n}Y_n(\gamma_{2\kappa}r)\right]-\right.$$
$$\left.-\frac{r\alpha\gamma_{2\kappa}}{\mu_{2\kappa}}\left[A_{5kn}J'_n(\gamma_{2\kappa}r)+A_{6kn}Y'_\kappa(\gamma_{2j}r)\right]\right\}\cos n\varphi e^{i(-\omega t+\gamma_p z)} \quad (9)$$

$$u_{\varphi\kappa}=-\sum_{n=0}^{\infty}\frac{n}{r}\left\{\gamma_\kappa\left[A_{1\kappa n}J_n(\gamma_{1\kappa}r)+A_{2\kappa n}Y_n(\gamma_{1\kappa}r)\right]+\frac{r\gamma_{2\kappa}}{n}\left[A_{3kn}J'_n(\gamma_{2k}r)+A_{4nk}Y'_n(\gamma_{2k}r)\right]-\right.$$
$$\left.-\frac{r}{\mu_{2k}}\left[A_{5kn}J_n(\gamma_{2k}r)+A_{6kn}Y_k(\gamma_{2k}r)\right]\right\}\sin n\varphi e^{i(-\omega t+\gamma_p z)};$$

$$u_{zk}=\sum_{n=0}^{\infty}\left\{\gamma_k\left[A_{1kn}J_n(\gamma_{1k}r)+A_{2kn}Y_n(\gamma_{1k}r)\right]+\frac{\gamma^2_{2k}}{n}\left[A_{5kn}J_n(\gamma_{2k}r)+A_{6kn}Y_n(\gamma_{2k}r)\right]\right\}\cos n\varphi e^{i(-\omega t+\gamma_p z)},$$

$$\gamma^2_{1k}=\bar{\mu}^2_{1k}-\gamma^2;\quad \bar{\mu}_{1k}=\frac{\omega}{a_{1k}\Gamma_k};\quad \gamma^2_{2k}=\bar{\mu}^2_{2k}-\gamma^2;$$

Where

$$\bar{\mu}_{2k}=\omega/c_{sk}\Gamma_{2k};\quad c^2_{sk}=\frac{\bar{\mu}_k}{\rho_k};\quad \gamma=m\pi/l,\ (m=1,2,...)\quad (k=1,2,3)$$

To determine arbitrary constants $A_{1kn},A_{2kn},A_{3kn},A_{4kn},A_{5kn},A_{6kn}$ boundary conditions are used (3). Then we get the system $6\kappa+3$ homogeneous algebraic equations with $6\kappa+3$ unknown. A necessary and sufficient condition for the existence of a solution to this system is that its determinant is equal to zero. The main determinant order $(6k+3\times 6k+3)$ and whose elements are expressed in terms of the n-th order Bessel and Neumann functions of the complex argument. Opening the determinant, we obtain the dispersion equation for the dissipative mechanical systems under consideration.

The dispersion characteristics are understood as the dependences of phase ($C=C_R+iC_I$) and group speeds $V=V_R+iV_I$ from wave number (γ_p)with various parameters of the mechanical system. It is known that the quantities C and V are related to the value of the root of the dispersion equation

$$\Delta(C,\omega,\gamma_p,\lambda)=0, \quad (10)$$

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Where ω- complex frequency; γ_p- wave number , λ – wavelength. The phase and group velocities are related to the value of the root of the dispersion equation by some complex dependencies.

Thus, in order to be able to calculate the dispersion characteristics, it is necessary to study the roots of equation (10) at the points of the complex plane, and also develop a method for their numerical determination. In the work, to solve the transcendental equation (10), the Mueller method is used, at each iteration of the Mueller method, the Gauss method is used with the main element.

Thus, the solution of equation (10) does not require disclosure of the determinant. As the initial approximation, we choose the phase velocities of the waves of the corresponding elastic system. For complex roots, the Mueller method simplifies calculations and provides faster convergence [22,23].

Consider the problem of the propagation of natural waves in a cylindrical shell at R_E=0 and $R_E \neq 0$. In each case, a dispersion equation is obtained, which is analyzed depending on the change in parameters. To obtain numerical results, based on the methods of Mueller, Gauss and Laplace, a program in the language C^{++}, and also used a software package MAPLE-18.

3. Axisymmetric viscoelastic waves in a single layer cylindrical shell

In this section, axisymmetric elastic waves in an extended viscoelastic cylindrical shell of circular cross section are considered for $h \to 0$. The equations of motion are expressed using the Lame equation, which describes viscoelastic waves in a homogeneous solid, and the imposition of boundary conditions of the "solid - vacuum" type on the front surfaces of the shell. Then the dispersion equation, without viscosity (R_E=0), takes the following form

$$4\gamma_p^2\left[k_2^2+(k_2^2-k_1^2)(k_2^2R^2-4)\right]-k_2^2\left[k_2^2(k_2^2R^2-4)+4k_1^2\right]=0,$$

And his decision can be expressed in the following elementary functions (for R_E=0):

$$C_{ph}(\omega)=\sqrt{\frac{E_{10}(R^2\omega^2\rho_0-E_{10})}{R^2\omega^2\rho_0^2(1-\nu^2)-\rho_0E_{10}}}. \qquad (11)$$

Where $k_1^2=\omega^2/c_p^2, k_2^2=\omega^2/c_s^2$, ρ_0- shell density, E_{10}- Young'a modulus.

An analysis of expression (11) shows that the frequency dependence function has two features at the points ω_1 and ω_2, which vanish, respectively, the numerator and denominator:

$$\omega_1=\sqrt{\frac{E_{10}}{R^2\rho_0}}, \omega_2=\sqrt{\frac{E_{10}}{R^2\rho_0(1-\nu^2)}}. \qquad (12)$$

From (11), it is also easy to obtain the asymptotics for the phase velocity C_{ph}:

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$$C_{ph}\underset{\omega=0}{} = \sqrt{\frac{E_{10}}{\rho_0}}, \qquad C_{ph}\underset{\omega\to\infty}{} = \sqrt{\frac{E_{10}}{\rho_0(1-\nu^2)}}. \qquad (13)$$

Since in any solid, the Poisson's ratio is within $0<\nu\leq 1/2$, inequality always holds $\omega_1<\omega_2$. It's obvious that $C_{ph}(\omega)$, and accordingly, the reciprocal of $\beta(\omega)$– the process propagation constant along the axis of the system is in the range $\omega_1<\omega<\omega_2$ turn out to be purely imaginary. This corresponds to waves that decay exponentially as the observation cross section moves away from the excitation cross section. Such waves are usually called non-propagating, since even in the absence of all dissipative losses they cannot fundamentally transfer the energy of the oscillatory process over a large distance.

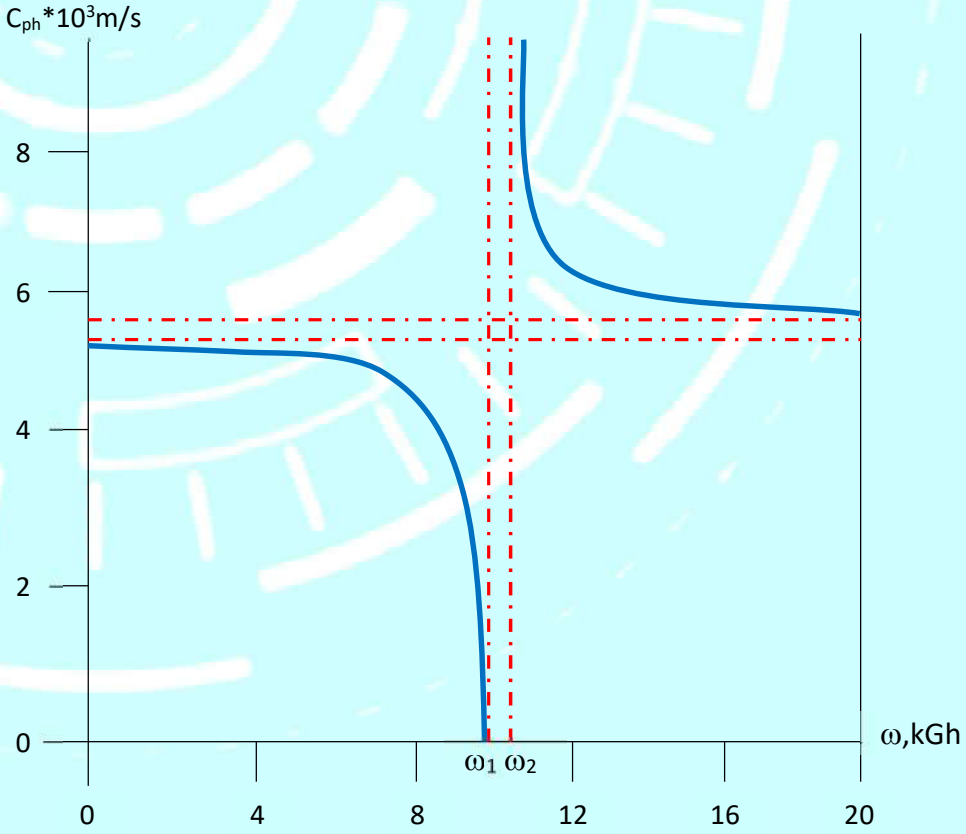


Figure 2. Dispersion curve for a cylindrical shell with a negligible steel wall thickness. The dashed-dotted line marks the boundaries of the non-proliferation bands. $\Delta f_{request}$ and impossible phase velocities (h = 0.01mm)

In this way, (ω_1,ω_2) it is a certain "forbidden band," or non-propagation band, the boundaries of which are indicated by a dash-dot line as an example of a frequency dependence constructed using formula (11) (Figure 2). The results were obtained with the following initial data: R=0.09м,ρ_0=7800кг/m^3,E=2.1 10^{11}N/m^3,ν=0.3.

From asymptotics for phase velocity, as well as from Fig. 2 and 3, it can be seen that in addition to the frequency range at which the oscillations cannot propagate along the thin-walled shell with concentrated excitation, there is also a region of impossible phase velocities. The values of the boundaries of the range of

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"forbidden" speeds are between the transverse and longitudinal speeds of sound: $C_s<C_{ph}(\omega=0)<C_{ph}(\omega\to\infty)<C_p$. From fig. 2 and 3, as well as from expression (11), it can be seen that $\omega = \omega_1$ at $C_{ph}= 0$, a $\omega = \omega_2$ at $C_{ph}\to\infty$.

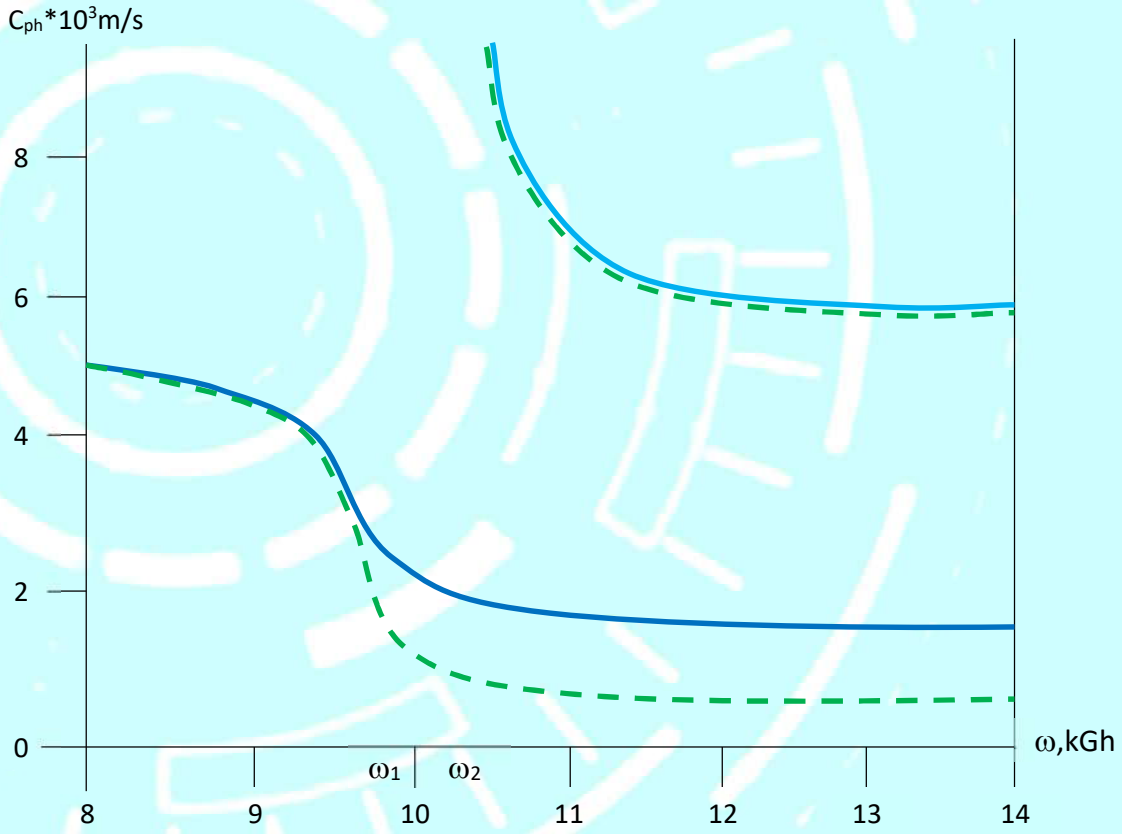


Figure 3. A family of dispersion curves obtained by numerically solving the initial equation for cylindrical shells with various wall thicknesses h = 11.5 (solid line); 1.0 mm (dotted)

The infinite value of the phase velocity of the wave propagating along the axis corresponds to the in-phase oscillation of the pipe along the entire length with the same frequency (natural frequency of oscillations of a thin-walled cylindrical shell), and its determining formula, which exactly corresponds to (11), but obtained in a different way, is given in some classical fundamental works, for example, in [24]. Less clear is the situation when the phase velocity is zero at a very specific non-zero frequency.

In this case, the entire shell also oscillates in phase, but at the same time it does not behave as a whole, but as an endless collection of independent, as if unconnected round rings. In other words, if the frequency ω_2 corresponds to the resonance of the tubes as a cylindrical whole ("tube" resonance), the frequency is resonant for the ring located in each of its cross sections.

Now we consider the problem of the propagation of free waves on a cylindrical shell, taking into account the viscous properties of the material. As a relaxation core, we use the fractional-exponential Rabotnov function [14]

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$$Э_\alpha(-\beta,t) = t^\alpha \sum_{n=0}^{\infty} \frac{(-\beta)^n t^{n(1+\alpha)}}{\Gamma((n+1)(1+\alpha))},$$

Where $\Gamma(n) = \int_0^\infty y^{n-1} e^{-y} dy$ – gamma function.

Then the dispersion equation takes the following form

$$(\Omega^2 - 2\tilde{\chi}^2)^2 J_0(a)J_1(b) + 4\tilde{\chi}^2 abJ_0(b)J_1(a) - 2\Omega^2 aJ_1(a)J_1(b) = 0, \quad (14)$$

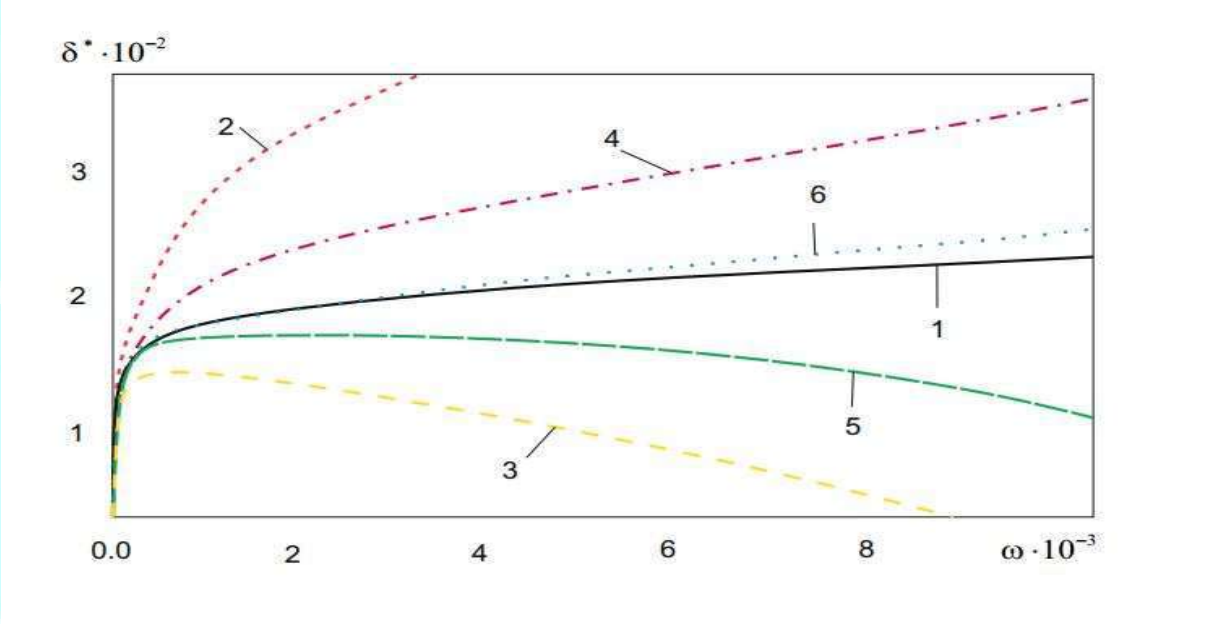


Figure 4. Change in the damping coefficient corresponding to the first mode versus frequency, at α=-3/4

where a and b are the inner and outer radii of the shell, $\tilde{\chi} = \chi - i\delta$, χ – wave number, $\delta > 0$ – attenuation coefficient,

$$\Omega^2 = \omega^2 \frac{1+\nu^F}{(1+\nu)E^F}, \quad E^F = 1 - \frac{k}{\beta + (i\omega)^{1+\alpha}},$$

$$\nu^F = \nu + \frac{1-2\nu}{2} \frac{k}{\beta + (i\omega)^{1+\alpha}},$$

Moreover, the unique branch of the power function $-\pi < \arg(i\omega) < \pi$, к,β-material parameters. The calculation results are shown in Figures 4 and 5. The change in the real parts of the phase velocities of the longitudinally radial vibrations depending on the frequency is almost the same, with the results without viscosity (Figs. 2 and 3). In the figure, the curve number corresponds to the number of terms in the expansion of the special functions of Bessel and Neumann. Results differ from each other up to 6%. The nature of the change in the imaginary parts (damping coefficient [25]) of the natural frequencies corresponding to the first mode turned out to be the same as in the elastic case (R_E=0).

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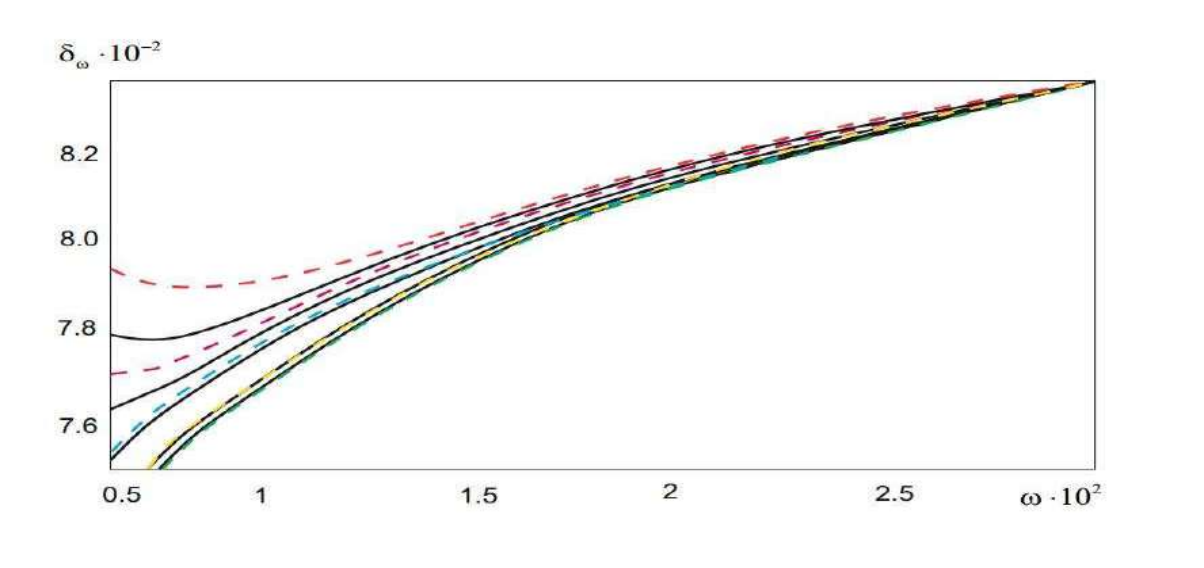


Figure 5. Change in damping coefficient corresponding to the first mode, versus frequency, at α=-1/6 (n=2,3,..)

The calculation results are shown in Fig. 4 and 5 at k = 0.5, β=1. In Figs. 4 and 5, solid lines show the numerical solutions of dispersion equation (14), and dashed lines show the asymptotics constructed in [26]. Based on numerical analysis, it was found that the behavior of the imaginary part of the phase velocity strongly depends on the singularity of the parameter α.

In the following example, we consider the change in the frequency of the shell depending on the half-thickness of the wall of the cylindrical shell. It can be seen from Fig. 6 that the resonance of a shell with thick walls is always at a frequency that corresponds to the resonance of a thin-walled shell of a smaller radius (a pipe with a thicker wall behaves like a narrower one). Dot $h= 2R$ or $\Delta =1$, Where $\Delta = h/2R$, corresponds to the disappearance of the inner volume of the shell and, thus, the transformation of the latter into a continuous homogeneous rod, the radius of which is equal to twice the radius of the original thin-walled shell. Thus, fig. 6. Demonstrates a smooth transition between the solutions of two classical problems (on the natural frequency of transverse vibrations of a thin-walled shell (h = 0) and a solid rod (h = 2R)), usually considered independently of each other.

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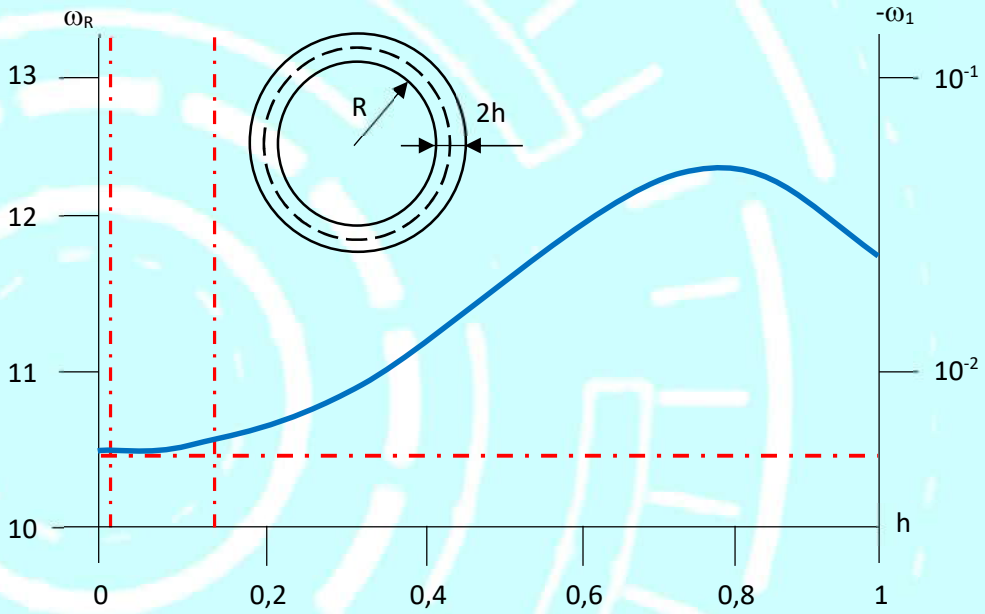


Fig. 6. The calculated dependence of the frequency of the "tube" resonance from the relative half-thickness of the wall of the cylindrical shell Δ

In the limit of the tendency of the wall thickness to zero ("thin-walled" approximation) the zero mode of the antisymmetric Lamb wave (“tube” wave), which is the only possible one in this case, is investigated, and an analytical expression for the dispersion dependence of the complex phase velocity on frequency is found. Taking into account the viscous properties of the material allows to reduce the phase velocity by 10%.

4. Wave propagation in three-layer cylindrical shells

An elastic shell with two symmetric viscoelastic coatings is considered. The equations of motion and the boundary conditions of a mechanical system satisfy system (1) - (3). The solution is also written in the form (9). Using the boundary (or contact) conditions, we obtain the dispersion equation (10), which is further solved numerically by the Mueller, Gauss, and Laplace methods.

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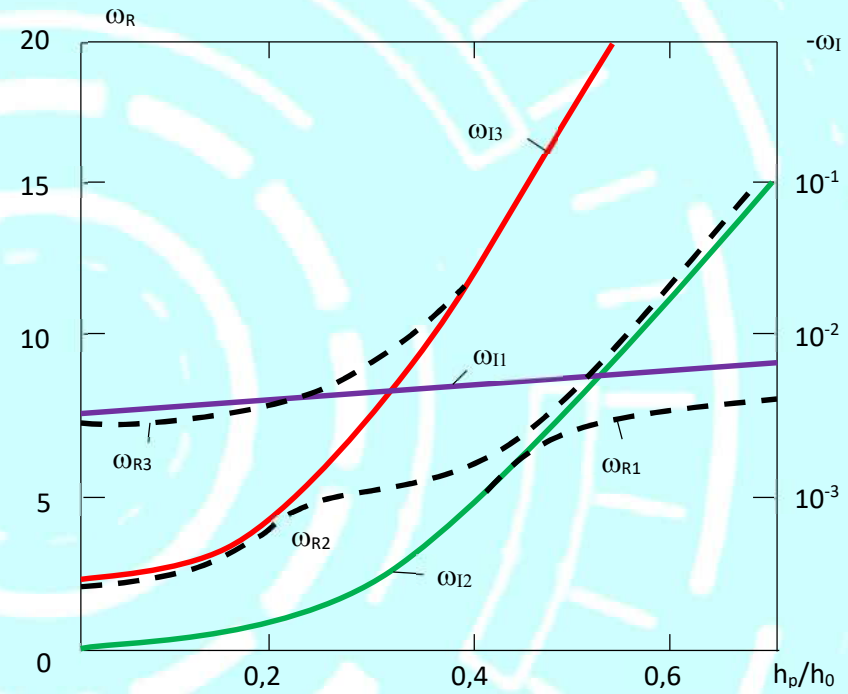


Fig. 7. The dependence of the frequencies and damping coefficient ω_l from h_p /h= C_0 for mechanical systems considered

The variation of the first two modes of the imaginary part of the complex frequency as a function of h_p /h_0 (h_0- shell thickness, h_p- coating thickness) of a dissipative-inhomogeneous mechanical system [27]. The calculation results are shown in Fig. 7.

Fig. 7 shows the dependence of the first two frequencies ω_R^1, ω_R^2 and corresponding damping factors ω_l^1, ω_l^2 from the value of the instantaneous stiffness of the shock absorber C. From the analysis of the graphs it follows that the dissipative properties of this system as a whole are determined not only by the rheology of its elements, but depend significantly on the interaction of their own vibration modes.

This effect is expressed in the fact that under certain conditions and up to a certain value of the stiffness of the shock absorber, the energetically more capacious (in this case, the second) form dissipates less energy than the less energy-intensive form. Then, starting with some value of the geometric parameter h_p /h=C_0 the process of energy dissipation by own forms is normalized and proceeds according to the energy hierarchy of forms. A real illustration of this effect is the intersection point of the damping coefficient graphs. ω_l^1 and ω_l^2 at $C = C^*$. And another feature: at this point, the difference in the attenuation rates of the two forms of vibration of the structure changes sign if h_p /h=C_0 (i.e. the system is structurally heterogeneous). As in the elastic problem, the eigenfrequencies ω_R^1, ω_R^2, which are slightly less than ω_1, ω_2, at this point approach. An analysis of problems of this type showed that the effect of the interaction of natural forms is observed only in

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structurally inhomogeneous systems (in this case, with elastic and viscoelastic elements) and with a noticeable approximation of the real parts of the natural frequencies ω_R^1, ω_R^2, and the imaginary parts of the frequencies (damping coefficients) change radically (sharply increases or decreases). The absence of at least one of these conditions excludes the manifestation of the effect. A physical explanation of the observed effect should be sought in the nature of the dynamic redistribution of energy of the system between two forms having the properties described above.

Conclusions

1. The mathematical formulation and methods for solving the problem of the propagation of natural waves on multilayer viscoelastic cylindrical shells are formulated.

2. The problems of the propagation of intrinsic waves on three-layer viscoelastic cylindrical shells are solved.

3. In the limit of the tendency of the wall thickness to zero ("thin-walled" approximation) the zero mode of the antisymmetric Lamb wave (“tube” wave), which is the only possible one in this case, is investigated, and an analytical expression for the dispersion dependence of the complex phase velocity on frequency is found.

4. To describe the dissipative properties of the system as a whole, the concept of "global damping coefficient" is introduced. In the case of a structurally homogeneous mechanical system, the global damping coefficient is entirely determined by the imaginary part of the first modular complex phase velocity. In the case of a structurally heterogeneous mechanical system, the imaginary parts of both the first and second natural frequencies act as the global damping coefficient, depending on the magnitude of the geometric parameters.

5. It was established that the optimal damping of oscillations of inhomogeneous systems will be when the real part of the phase velocity of the various modes is close (or approaching), and the corresponding imaginary parts of these phase velocities intersect at this point (or are equal to each other). In this case, both vibration modes of the mechanical system provide the same energy dissipation.

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