MATHEMATICAL MODELING AND OPTIMIZATION OF RESOURCE ALLOCATION IN CLOUD COMPUTING ENVIRONMENTS

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Abstract. Cloud computing systems need smart and effective ways to allocate resources since workloads are different, user needs are always changing, and service-level agreements (SLAs) are rigorous. Bad allocation of resources leads to slower performance, higher energy use, and less efficient use of computing resources. This research presents a thorough mathematical modeling and multi-objective optimization methodology for resource allocation in cloud computing systems. The allocation problem is structured as a restricted multi-dimensional optimization model that takes into account CPU, memory, bandwidth, storage, execution time, energy consumption, operational cost, and utilization. To solve the model quickly, a combination of a genetic algorithm and a local search optimization approach is used. Experiments using CloudSim, statistical analysis, and graphical assessments show that the suggested strategy works far better than traditional methods in terms of system performance, energy efficiency, and SLA compliance. The results show that mathematical optimization is a strong and flexible basis for smart cloud resource management in big computing systems.

Keywords: Cloud computing, resource allocation, mathematical modeling, optimization, and cloud scheduling.

Аннотация. Системы облачных вычислений нуждаются в интеллектуальных и эффективных способах распределения ресурсов, поскольку рабочие нагрузки различны, потребности пользователей постоянно меняются, а соглашения об уровне обслуживания (SLA) являются строгими. Неправильное распределение ресурсов приводит к снижению производительности, увеличению энергопотребления и менее эффективному использованию вычислительных ресурсов. В данном исследовании представлена подробная математическая модель и методология многоцелевой оптимизации для распределения ресурсов в системах облачных вычислений. Задача распределения структурирована как ограниченная многомерная оптимизационная модель, учитывающая ЦП, память, пропускную способность, хранилище, время выполнения,

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энергопотребление, эксплуатационные расходы и использование. Для быстрого решения модели используется комбинация генетического алгоритма и подхода локального поиска. Эксперименты с использованием CloudSim, статистический анализ и графические оценки показывают, что предложенная стратегия работает значительно лучше традиционных методов с точки зрения производительности системы, энергоэффективности и соответствия SLA. Результаты показывают, что математическая оптимизация является прочной и гибкой основой для интеллектуального управления облачными ресурсами в больших вычислительных системах.

Ключевые слова: облачные вычисления, распределение ресурсов, математическое моделирование, оптимизация и планирование облачных задач.

Annotatsiya. Bulutli hisoblash tizimlari resurslarni taqsimlashning aqlli va samarali usullariga muhtoj, chunki ish yuklamalari har xil, foydalanuvchi ehtiyojlari doimo o'zgarib turadi va xizmat ko'rsatish darajasidagi kelishuvlar (SLA) qat'iydir. Resurslarning noto'g'ri taqsimlanishi sekinroq ishlashga, yuqori energiya sarfiga va hisoblash resurslaridan samarasiz foydalanishga olib keladi. Ushbu tadqiqot bulutli hisoblash tizimlarida resurslarni taqsimlash uchun batafsil matematik modellashtirish va ko'p maqsadli optimallashtirish metodologiyasini taqdim etadi. Taqsimlash muammosi protsessor, xotira, o'tkazish qobiliyati, saqlash, bajarish vaqti, energiya sarfi, operatsion xarajatlar va foydalanishni hisobga oladigan cheklangan ko'p o'lchovli optimallashtirish modeli sifatida tuzilgan. Modelni tezda hal qilish uchun genetik algoritm va mahalliy qidiruv optimallashtirish yondashuvining kombinatsiyasi qo'llaniladi. CloudSim yordamida o'tkazilgan tajribalar, statistik tahlil va grafik baholashlar shuni ko'rsatadiki, taklif qilingan strategiya tizim ishlashi, energiya samaradorligi va SLA muvofiqligi nuqtai nazaridan an'anaviy usullarga qaraganda ancha yaxshi ishlaydi. Natijalar shuni ko'rsatadiki, matematik optimallashtirish katta hisoblash tizimlarida aqlli bulut resurslarini boshqarish uchun kuchli va moslashuvchan asosdir.

Kalit so'zlar: Bulutli hisoblash, resurslarni taqsimlash, matematik modellashtirish, optimallashtirish va bulutli jadval tuzish

1. INTRODUCTION

Cloud computing is now the backbone of modern information systems. It supports important services like artificial intelligence, big data analytics, the Internet of Things (IoT), and smart city platforms. Cloud infrastructures are made up of large-scale distributed computing resources that need to be managed well to work well and be reliable.

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One of the hardest things to do in cloud settings is resource allocation, which decides how virtual machines (VMs) or containers are placed on real servers. When servers are not allocated correctly, they become overloaded, underused, have high latency, use more energy, and break SLAs more often.

Most of the ways that resources are now allocated are heuristic and reactive. They are easy to compute, but they don't have any theoretical guarantees and can't handle changing workloads well. Methods based on machine learning make systems more flexible, but they need a lot of training data and are hard to understand.

Mathematical modeling gives us a formal way to describe cloud systems, while optimization theory helps us make decisions in a planned and methodical way. This paper concentrates on establishing a mathematical and optimization-driven resource allocation paradigm for cloud computing environments.

The primary contributions of this study are:

- Creation of a structured mathematical model for the distribution of cloud resources.
- Creating a multi-objective restricted optimization problem.
- Designing a hybrid optimization method.
- A lot of testing and simulation.

2. . Work that is related

There are four primary groups of studies on cloud resource allocation that have come before:

2.1 Heuristic Techniques

Round Robin, First Fit, and Best Fit are common, however they don't take into account different types of labor or the fact that there are limits on how many resources may be used at once.

2.2 Metaheuristic Approaches

Genetic Algorithm, Particle Swarm Optimization, and Ant Colony Optimization enhance solution quality, although frequently encounter significant computing expenses and parameter sensitivity.

2.3 Techniques for Machine Learning

Deep learning and reinforcement learning are flexible, but they need a lot of training data and don't have any guarantees of being the best way to do things.

2.4 Ways to Optimize Math

Linear and integer programming models have strong theoretical bases, but they are quite expensive to run on big cloud systems.

This research amalgamates mathematical modeling with hybrid optimization to attain both theoretical precision and actual efficacy.

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3. Model of the Cloud System

Let

$$S=\{s_1, s_2,\dots s_m\}(1)$$

be the group of physical servers, and

$$V=\{v_1, v_2,\dots v_n\}(2)$$

be the group of VM requests.

Each server can hold:

$$(C_i, M_i, B_i, D_i)(3)$$

Every VM needs:

$$(c_j, m_j, b_j, d_j)(4)$$

Decision variable:

$$x_{ij}=\begin{cases}1 & \text{if VM } v_j \text{ is assigned to server } s_i,(5)\\ 0 & \text{Otherwise}\end{cases}$$

4. Constraints

CPU:

$$\sum_{j=1}^{n} c_j\, x_{ij}\le C_i(6)$$

Memory:

$$\sum_{j=1}^{n} m_j\, x_{ij}\le M_i(7)$$

Bandwidth:

$$\sum_{j=1}^{n} b_j\, x_{ij}\le B_i(8)$$

Storage:

$$\sum_{j=1}^{n} d_j\, x_{ij}\le D_i(9)$$

Assignment:

$$\sum_{i=1}^{m} d_j\, x_{ij}\le D_i(10)$$

Binary condition:

$$x_{ij}\in\{0,1\}(11)$$

5. Functions of Objective

Minimization of performance:

$$F_1=\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}\, x_{ij}(12)$$

Minimizing energy:

$$F_2=\sum_{i=1}^{m}(P_i^{idle}+P_i^{load}u_i)(13)$$

Minimizing costs:

$$F_3=\sum_{i=1}^{m}\sum_{j=1}^{n} cost_{ij}\, x_{ij}(14)$$

Maximizing use:

$$F_4=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} cost_{ij}x_{ij}}{\sum_{i=1}^{m} c_i}(15)$$

The main goal is:

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$$\min F = w_1 F_1 + w_2 F_2 + w_3 F_3 - w_4 F_4$$

6. Algorithm for Optimization

We use a hybrid genetic algorithm with local search (GA–LS).

Pseudocode:

Initialize population
Evaluate fitness
While termination condition not met:
  Selection
  Crossover
  Mutation
  Local Search
  Update population
Return the best answer

This method makes sure that exploration happens all over the world and that things are improved in specific places.

7. Setting Up the Experiment

Simulator: CloudSim
Servers: 100
VMs: 1000
Metrics:

- Time to execute
- Energy use
- Using resources
- Rate of SLA violations
- Index of load balance

The same experiment was done 30 times.

8. Results

Method	Exec. Time	Energy	Utilization	SLA
Round Robin	335	21.5	68.9	9.3
Best Fit	297	19.2	73.4	7.1
ACO	268	17.4	78.6	5.4
Proposed	**229**	**14.6**	**86.3**	**2.9**

9. Statistical Analysis

The paired t-test showed that $p < 0.01$ for all metrics, which means that the improvement was statistically significant.

10. Talk

Due to balanced resource distribution, the suggested model has higher utilization, uses less energy, and has fewer SLA violations. The hybrid

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optimization makes sure that the solution converges quickly, while the mathematical formulation makes sure that it is possible and stable.

The suggested strategy is explainable, predictable, and doesn't depend on training datasets, unlike learning-based methods.

11. Analysis of Scalability

Tests with more VMs show almost linear scalability, which shows that it is good for big cloud systems.

12. Threats to Validity

The primary constraint is simulation-based assessment. Real-world cloud deployment should be part of future work.

13. Conclusion

This study put up a methodology for mathematical modeling and optimization for allocating resources in the cloud.The hybrid optimization technique greatly boosts performance, energy efficiency, use, and SLA compliance. The findings validate that mathematical optimization serves as a robust foundation for intelligent cloud resource management.

14. Work to Come

- Adaptive optimization in real time
- Integration of edge and cloud
- Scheduling that takes carbon into account
- Cloud environments that work together

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