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INTUITIONISTIC FUZZY AGGREGATION OPERATORS IN MULTI-CRITERIA DECISION-MAKING

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Annotation. This paper explores the application of intuitionistic fuzzy sets (IFS) and aggregation operators in solving multi-criteria decision-making (MCDM) problems. The study focuses on the use of the intuitionistic fuzzy Choquet integral operator, which effectively handles the interdependencies among criteria while aggregating linguistic evaluations expressed as intuitionistic fuzzy values. Through an illustrative example of supplier selection based on machine competencies, the paper demonstrates how partial evaluations of alternatives are represented using membership, non-membership, and hesitancy degrees. The ranking of alternatives is achieved using score and accuracy functions, ensuring a robust decision-making process. This work highlights the significance of IFS-based aggregation operators in addressing uncertainty and vagueness in real-world decision scenarios.

Key words: intuitionistic fuzzy set, agregation operator, design making, multi-criteria design-making, Choquet integral, Sugeno integral, ordered weighted averaging.

Аннотация. В статье рассматривается применение интуиционистских нечётких множеств (ИНМ) и агрегирующих операторов для решения задач многокритериального принятия решений (МКПР). Основное внимание уделено использованию интуиционистского нечёткого интегрального оператора Шоке, который эффективно учитывает взаимосвязи между критериями при агрегировании лингвистических оценок, выраженных интуиционистскими нечёткими значениями. На примере задачи выбора поставщика на основе компетенций в области оборудования продемонстрировано, как частные оценки альтернатив представлены с помощью степеней принадлежности, непринадлежности и неопределённости (колебания). Ранжирование альтернатив осуществляется на основе функций

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оценки (score) и точности (accuracy), обеспечивая надёжность и устойчивость процесса принятия решений. Работа подчёркивает значимость агрегирующих операторов на основе ИНМ для управления неопределённостью и нечёткостью в реальных задачах принятия решений.
Ключевые слова: интуиционистские нечёткие множества, агрегирующие операторы, принятие решений, многокритериальное принятие решений, интеграл Шоке, интеграл Сугено, упорядоченное взвешенное усреднение
Annotatsiya. Mazkur maqolada ko‘p mezonli qaror qabul qilish (KMQ) masalalarini yechishda intuitsionistik noravshan to‘plamlar (INT) hamda agregatsiya operatorlaridan foydalanish imkoniyatlari tadqiq etiladi. Asosiy e’tibor mezonlararo o‘zaro bog‘liqliklarni samarali hisobga oluvchi intuitsionistik noravshan Choquet integral operatoriga qaratilgan bo‘lib, u lingvistik baholash natijalarini intuitsionistik noravshan qiymatlar ko‘rinishida ifodalash imkonini beradi. Uskunalarning kompetensiyalariga asoslanib yetkazib beruvchini tanlash masalasi misolida alternativlarning qisman baholari tegishlilik, tegishli emaslik va noaniqlik (ikkilanish) darajalari yordamida qanday aks ettirilishi ko‘rsatib berilgan. Alternativalarni saralash score (baholash) va accuracy (aniqlik) funksiyalariga asoslangan holda amalga oshiriladi, bu esa qaror qabul qilish jarayonining ishonchliligi va barqarorligini ta’minlaydi. Ushbu tadqiqot real qaror qabul qilish vaziyatlaridagi noaniqlik va mujmallikni boshqarishda INT asosidagi agregatsiya operatorlarining muhim ahamiyatini ta’kidlaydi.
Kalit so'zlar: intuitsionistik noravshan to‘plam, agregatsiya operatori, qaror qabul qilish, ko‘p mezonli qaror qabul qilish, Choquet integrali, Sugeno integrali, tartiblangan vaznli o‘rtacha.

INTRODUCTION

Decision-making processes often involve multiple criteria, where alternatives must be evaluated based on various attributes or factors. In many practical situations, such evaluations are inherently uncertain and imprecise due to the subjective nature of human judgment. To address this challenge, Zadeh's fuzzy set theory has been widely adopted, but it only considers membership degrees. Atanassov's intuitionistic fuzzy sets (IFS), which incorporate both membership and non-membership degrees alongside a hesitancy margin, provide a more comprehensive framework for modeling uncertainty [1].

Aggregation operators play a pivotal role in combining these evaluations into a unified result that reflects the overall performance of each alternative. Traditional aggregation operators like arithmetic mean, weighted sum, and ordered weighted averaging (OWA) have been extended to fuzzy environments. However, when

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criteria exhibit interdependencies, advanced tools such as fuzzy integrals – particularly the Choquet integral – prove indispensable.

This paper aims to demonstrate the effectiveness of intuitionistic fuzzy Choquet integral operators in solving MCDM problems. We present a step-by-step methodology, including the formation of an intuitionistic fuzzy decision matrix, reordering based on score functions, and aggregation using the Choquet integral. An example of supplier selection is provided to illustrate the practical applicability of the proposed approach. By integrating IFS with the Choquet integral, this study offers a robust solution for decision-makers operating in complex and uncertain environments.

Aggregation operators are mathematical functions that combine different pieces of information. They aggregate all the objects into a single object of a same set so that the final result of aggregation takes into account all the individual values.

These are generally used in computational intelligence where linguistically expressed pieces of information are used together. Some well-known examples are arithmetic mean [2], weighted minimum and maximum [3], weighted sum [4], median [5], and ordered weighted averaging (OWA) operators [6]. Also, fuzzy integrals such as Choquet and Sugeno integrals are used as aggregation operator. We will discuss some of the well-known fuzzy aggregating operators.

METHODS AND RESULTS

Multi-criteria decision-making problem is a method for finding the best alternative among all the alternatives evaluated using a set of attributes/criteria. Alternatives are evaluated on the basis of criterion or attribute. Let $B = \{b_1, b_2, b_3, \dots, b_m\}$ be a set of attributes and $C = \{c_1, c_2, c_3, \dots, c_n\}$ be a set of alternatives. Partial evaluation of the alternatives, $c_i (i = 1,2,3, \dots, n)$, is carried out with respect to the attributes or criteria, $b_j (j = 1,2,3, \dots, m)$. Partial evaluation, c_{ij}, is expressed using intuitionistic fuzzy values, $c_{ij} = (\mu_{ij}, \nu_{ij})$, where μ_{ij} is the satisfaction degree which means that c_i satisfies the criterion b_j and ν_{ij} is the dissatisfaction degree which means that c_i does not satisfy the criterion b_j with the condition $0 \leq \mu_{ij} \leq 1,\ 0 \leq \nu_{ij} \leq 1$. A multi-attribute decision-making problem is expressed in matrix form:

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$$D = \begin{bmatrix} & b_1 & b_2 & b_3 & \dots & b_m \\ c_1 & (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & (\mu_{13}, \nu_{13}) & \dots & (\mu_{1n}, \nu_{1m}) \\ c_2 & (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & (\mu_{23}, \nu_{23}) & \dots & (\mu_{2n}, \nu_{2m}) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots \\ c_n & (\mu_{n1}, \nu_{n1}) & (\mu_{n2}, \nu_{n2}) & (\mu_{n3}, \nu_{n3}) & \dots & (\mu_{nm}, \nu_{nm}) \end{bmatrix}$$

Score function, $S(c_{ij})$, of the partial evaluation c_{ij} of the alternative c_i is evaluated to rank c_{ij}. If there is no difference between the two score functions, then accuracy function, $H(c_{ij})$, is used to rank c_{ij} based on the accuracy.

Example of decision-making problem to find an expert supplier based on supplier's competencies in making a machine is given. Suppose there are three suppliers and their ability is judged by the attributes of the machine such as (i) innovative level, (ii) longevity, and (iii) cost. We are to take a decision, which supplier is to be selected for ordering a machine. Attributes are denoted as (b_1, b_2, b_2) and the three suppliers, which are the alternatives, are denoted as (c_1, c_2, c_3). To evaluate the competencies of the experts, 10 candidates are invited. Suppose there are six candidates who judge the attribute b_1 of the expert c_1 as strong and other three candidates who judge the attribute b_1 of the expert c_1 as not strong and the remaining one candidate does not judge the candidate as strong or not strong. Then, evaluating value of the attribute b_1 of c_1 may be expressed using an intuitionistic fuzzy value $c_{ij} = (0.6, 0.3)$, where c_{ij} is the partial evaluation of any alternatives, c_i with respect to attributes, b_j. Likewise, the results of 10 candidates to the 3 experts according to the 3 criteria (attributes) together are performed and an intuitionistic fuzzy decision matrix of the experts is formed:

	b_1	b_2	b_3
c_1	$(0.6, 0.3)$	$(0.5, 0.4)$	$(0.7, 0.2)$
c_2	$(0.4, 0.3)$	$(0.6, 0.2)$	$(0.5, 0.2)$
c_3	$(0.4, 0.3)$	$(0.3, 0.5)$	$(0.6, 0.3)$

Partial evaluation c_{ij} of candidate supplier c_i with respect to the attributes or criteria, b_j, is reordered such that $c_{i(j)} \leq c_{i(j+1)}$.

Reordering is done on the basis of score function [6,7].

As has been said, the partial evaluation of the alternatives c_i with respect to the attributes b_j is made by intuitionistic fuzzy values $c_{ij} = (\mu_{ij}, \nu_{ij})$ and the decision matrix (D) is formed:

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$$D = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

Then, score on c_{ij} is given as:

$$S(c_{ij}) = \mu_{c(ij)} - \nu_{c(ij)} \quad \text{and} \quad S(c) \in [-1,1].$$

It represents the difference of membership and non-membership values. If the score values are similar, the accuracy degree is evaluated.

$$H(c_{ij}) = \mu_{c_{ij}} + \nu_{c_{ij}}, \text{ where } H(c) \in [0,1].$$

After reordering, which is done on the basis of score function $c_{i(j)} \leq c_{i(j+1)}$, we get:

$$c_{1(1)} = (0.5,0.4),\ c_{1(2)} = (0.6,0.3), c_{1(3)} = (0.7,0.2)$$
$$c_{2(1)} = (0.4,0.3), c_{2(2)} = (0.5,0.2), c_{2(3)} = (0.6,0.2)$$
$$c_{3(1)} = (0.3,0.5), c_{3(2)} = (0.4,0.3), c_{3(3)} = (0.6,0.3)$$

Let the fuzzy measure of the criterion b_1, b_2, b_3 or group of criteria i.e, the importance of each criterion, be given as:

$$\mu(b_1) = 0.4, \qquad \mu(b_2) = 0.2, \qquad \mu(b_3) = 0.3.$$

Using equation

$$\lambda + 1 = \prod_{i=1}^{n}(1 + \lambda g_i),$$

we obtain the value of λ and we get,

$$\mu(b_1, b_2) = 0.63,\ \mu(b_2, b_3) = 0.52,\ \mu(b_1, b_3) = 0.7445,\ \mu(b_1, b_2, b_3) = 1,$$

where μ is a fuzzy measure.

Using intuitionistic fuzzy Choquet integral operator:

$$c_i = IFC_\mu(c_{i1}, c_{i2}, c_{i3}, \dots, c_{in})$$
$$= \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{c_{i(j)}}\right)^{\mu(A_j)-\mu(A_{j+1})}, \prod_{j=1}^{n} \nu_{c_{i(j)}}^{\mu(A_j)-\mu(A_{j+1})}\right)$$

where $A_{(j)} = \{b_{(j)}, \dots, b_{(3)}\}$, $A_{(3+1)} = \emptyset$. We are to aggregate c_{ij} corresponding to the supplier $c_i (i = 1,2,3)$.

For computing c_1, we require to compute $\mu(A_{(j)})$.

We have $c_{1(1)} = (0.5,\ 0.4)$, $c_{1(2)} = (0.6,\ 0.3)$, $c_{1(3)} = (0.7,\ 0.2)$.

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So, $(1) = 2, (2) = 1, (3) = 3.$

Hence, we get

$$A_{(1)} = \{b_{(1)}, b_{(2)}, b_{(3)}\} = \{b_2, b_1, b_3\} \text{ and } \mu(A_{(1)}) = 1,$$
$$A_{(2)} = \{b_{(2)}, b_{(3)}\} = \{b_1, b_3\} \text{ and } \mu(A_{(2)}) = 0.7445,$$
$$A_{(3)} = \{b_{(3)}\} = \{b_3\} \text{ and } \mu(A_{(3)}) = 0.3.$$

$$c_1 = IFC_\mu(c_{11}, c_{12}, c_{13})$$
$$= \left(1 - \prod_{j=1}^{3} \left(1 - \mu_{c_{1(j)}}\right)^{\mu(A_{(j)}) - \mu(A_{(j+1)})}, \prod_{j=1}^{3} \nu_{c_{1(j)}}^{\mu(A_{(j)}) - \mu(A_{(j+1)})}\right)$$

$$\Rightarrow c_1 = [(1 - (1 - 0.5)^{1-0.7445} \times (1 - 0.6)^{0.7445 - .3} \times (1 - 0.7)^{0.3},$$
$$0.4^{1-0.7445} \times 0.3^{0.7445 - .33} \times 0.2^{0.3})]$$
$$= (1 - 0.5^{0.255} \times 0.4^{0.4445} \times 0.3^{0.3}, \quad 0.4^{0.255} \times 0.3^{0.4445} \times 0.2^{0.3})$$
$$= (0.6115, \quad 0.2964)$$

Likewise, for computing c_2, we have

$$c_{2(1)} = (0.4, 0.3), \qquad c_{2(2)} = (0.5, 0.2), \qquad c_{2(3)} = (0.6, 0.2)$$

Proceeding as above, we obtain

$$A_{(1)} = = \{b_{(1)}, b_{(2)}, b_{(3)}\} = \{b_1, b_3, b_2\}, \quad \mu(A_1) = 1,$$
$$A_{(2)} = \{b_{(2)}, b_{(3)}\} = \{b_3, b_2\}, \quad \mu(A_2) = 0.52,$$
$$A_{(3)} = \{b_{(3)}\} = \{b_2\}, \quad \mu(A_3) = 0.2.$$

$$c_2 = \left(1 - \prod_{j=1}^{4} \left(1 - \mu_{c_{2(j)}}\right)^{\mu(A_{(j)}) - \mu(A_{(j+1)})}, \prod_{j=1}^{4} \nu_{c_{2(j)}}^{\mu(A_{(j)}) - \mu(A_{(j+1)})}\right)$$
$$= [(1$$
$$- 0.6^{1-0.52} \times 0.5^{0.52-0.2} \times 0.4^{0.2}, 0.3^{1-0.52} \times 0.2^{0.52-0.2} \times 0.2^{0.2})]$$
$$= (0.4781, 0.2430)$$

Likewise, for computing c_3, we have

$$c_{3(1)} = (0.3, 0.5), \qquad c_{3(2)} = (0.4, 0.3), \qquad c_{3(3)} = (0.6, 0.3)$$

Thus, we get

$$A_{(1)} = \{b_{(1)}, b_{(2)}, b_{(3)}\} = \{b_2, b_1, b_3\}, \qquad \mu(A_1) = 1$$
$$A_{(2)} = \{b_{(2)}, b_{(3)}\} = \{b_1, b_3\}, \qquad \mu(A_2) = 0.7445$$

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$$A_{(3)} = \{b_{(3)}\} = \{b_3\}, \qquad \mu(A_3) = 0.3$$

$$c_3 = \left(1 - \prod_{j=1}^{4} \left(1 - \mu_{c_{3(j)}}\right)^{\mu(A_{(j)}) - \mu(A_{(j+1)})}, \quad \prod_{j=1}^{4} \nu_{c_{3(j)}}^{\mu(A_{(j)}) - \mu(A_{(j+1)})}\right)$$
$$= (1 - (1 - 0.3)^{1-0.7445} \times (1 - 0.4)^{0.7445-0.3} \times (1 - 0.6)^{0.3},$$
$$0.5^{1-0.7445} \times 0.3^{0.7445-\ .3} \times 0.3^{0.3}) = 0.4474, \quad 0.3418$$

So, $c_1 = (0.6115, 0.2964)$, $c_2 = (0.4781, 0.2430)$, $c_3 = (0.4474, 0.3418)$

Using the score function (difference between membership and non-membership values), ranking is done as: $c_1 > c_2 > c_3$.

Thus, c_1 supplier is the best.

CONCLUSION

The application of intuitionistic fuzzy sets and aggregation operators provides a powerful framework for addressing multi-criteria decision-making problems under uncertainty. This paper has demonstrated the utility of the intuitionistic fuzzy Choquet integral operator in aggregating evaluations while accounting for interdependencies among criteria. The example of supplier selection highlighted the systematic process of forming an intuitionistic fuzzy decision matrix, reordering alternatives based on score and accuracy functions, and determining the optimal choice through aggregation.

The results underscore the importance of considering both membership and non-membership degrees in decision-making, as well as the need to account for interactions among criteria. The proposed methodology not only enhances the precision of decisions but also accommodates the inherent subjectivity and ambiguity in human judgments. Future research could extend this approach to larger datasets or explore its integration with other advanced fuzzy tools, further expanding its applicability in diverse domains such as supply chain management, healthcare, and environmental planning.

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