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SYNTHESIS OF HYBRID RLS–NEURAL NETWORK-BASED ADAPTIVE DISCRETE CONTROLLERS FOR NONLINEAR SYSTEMS

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Abstract. Parameter variations and unmodeled nonlinearities significantly degrade the performance of conventional fixed-parameter controllers in digital industrial systems. To address this, a hybrid adaptive discrete control algorithm for uncertain nonlinear processes is proposed. The approach combines recursive least squares estimation with a forgetting factor for real-time parameter adaptation and a parallel neural network for compensating nonlinear disturbances and modeling errors. Both components are integrated within a certainty equivalence control framework, where control inputs are computed using updated parameter estimates and neural weights are adjusted via a gradient-based learning rule. Lyapunov-based analysis guarantees boundedness of tracking and estimation errors under practical conditions of parameter variations and disturbances. The proposed method is computationally efficient and suitable for real-time implementation in robotic, aerospace, and industrial process control systems.

Keywords: adaptive discrete control, recursive least squares, neural network compensation, parameter uncertainty, Lyapunov stability

Annotatsiya. Parametrlarning o‘zgarishi va modellanmagan nochiziqliklar raqamli sanoat tizimlarida an’anaviy qat’iy parametrli regulyatorlar samaradorligini sezilarli darajada pasaytiradi. Ushbu muammoni hal qilish uchun noaniq nochiziqli tizimlar uchun gibrid adaptiv diskret boshqaruv algoritmi taklif etiladi. Taklif etilgan yondashuvda real vaqt rejimida parametrlarni baholash uchun unutish koeffitsientli rekursiv eng kichik kvadratlar (RLS) usuli hamda modellash xatolari va nochiziqli buzilishlarni kompensatsiya qilish uchun parallel neyron tarmoq integratsiyalangan. Boshqaruv signali aniqlik ekvivalentligi printsipi asosida shakllantiriladi, bunda parametr baholari asosiy modelni moslashtiradi, neyron tarmoq esa qoldiq nochiziqliklarni kompensatsiya qiladi. Neyron tarmoq og‘irliklari kuzatuv xatosiga asoslangan gradient algoritmi orqali yangilanadi. Lyapunov funksiyasi asosida o‘tkazilgan tahlil parametrlar o‘zgarishi va tashqi

ISSN 3030-3907

buzilishlar mavjud sharoitlarda tizimning barqarorligini hamda kuzatuv va baholash xatolarining chegaralanganligini kafolatlaydi. Taklif etilayotgan algoritm hisoblash jihatidan samarali bo‘lib, raqamli boshqaruv qurilmalarida real vaqt rejimida amalga oshirishga mos keladi hamda sanoat texnologik jarayonlar va robototexnika tizimlarda qo‘llash imkonini beradi.

Kalit so‘zlar: adaptiv diskret boshqaruv, gibrid adaptiv boshqaruv, rekursiv eng kichik kvadratlar (RLS), neyron tarmoq asosidagi kompensatsiya, parametr noaniqligi, nochiziqli tizimlar, Lyapunov barqarorligi, real vaqt rejimida amalga oshirish.

Аннотация. Изменения параметров и немоделированные нелинейности существенно снижают эффективность традиционных регуляторов с фиксированными параметрами в цифровых промышленных системах. Для решения данной проблемы предлагается гибридный адаптивный алгоритм дискретного управления для неопределённых нелинейных процессов. Предлагаемый подход объединяет метод рекурсивных наименьших квадратов с коэффициентом забывания для адаптации параметров в реальном времени и параллельную нейронную сеть для компенсации нелинейных возмущений и ошибок моделирования. Оба компонента интегрированы в рамках принципа управления с эквивалентностью определённости, при котором управляющее воздействие формируется на основе обновлённых оценок параметров, а веса нейронной сети настраиваются с использованием градиентного алгоритма обучения. Анализ на основе функции Ляпунова гарантирует ограниченность ошибок слежения и оценивания в условиях изменения параметров и внешних возмущений, характерных для практических задач. Предложенный метод обладает низкой вычислительной сложностью и пригоден для реализации в реальном времени в робототехнических, аэрокосмических и промышленных системах управления.

Ключевые слова: адаптивное дискретное управление, рекурсивные наименьшие квадраты, нейросетевая компенсация, параметрическая неопределённость, устойчивость по Ляпунову

INTRODUCTION.

Real industrial systems operate under uncertainty and time-varying conditions, where parameters drift due to environmental changes, nonlinearities, and component aging. Fixed-parameter controllers designed for nominal models often fail, causing oscillations or slow response.

Adaptive control addresses this by updating controller parameters online based on measured data. Recursive least squares (RLS) is widely used due to its

ISSN 3030-3907

computational efficiency and suitability for real-time implementation, and with a forgetting factor it can track slowly varying parameters.

However, RLS-based methods are limited in the presence of nonlinear disturbances and unmodeled dynamics, which reduces estimation accuracy and control performance. Neural networks can compensate for these nonlinear effects by approximating residual dynamics.

This paper proposes a hybrid adaptive discrete control algorithm combining RLS estimation and neural network compensation. The control law is based on the certainty equivalence principle, and stability is analyzed using a Lyapunov approach.

The proposed method ensures bounded signals and improved tracking performance under uncertainty and nonlinear disturbances.

Problem formulation. Consider a discrete-time dynamic system with uncertain parameters and external disturbances. In practical applications, the plant model is often not precisely known, and its parameters may vary due to environmental changes, load variations, nonlinearities, and modeling errors. Therefore, it is convenient to represent the system in a linear-in-parameters regression form for adaptive identification and control design.

Let the input–output behavior of the plant be described by the following discrete-time model

$$y(k) = \varphi^T(k)\theta(k) + v(k) \quad (1)$$

where $(y(k))$ is the measured system output at the discrete time instant k, $\varphi(k)$ is the regressor vector constructed from past values of the input and output signals, $(\theta(k))$ is the vector of unknown or slowly time-varying parameters of the system, and $v(k)$ represents disturbances, measurement noise, and unmodeled dynamics. The regressor vector typically contains delayed inputs and outputs of the system and can be written in the form

$$\varphi(k) = [-y(k-1), -y(k-2), \dots, -y(k-n), u(k-1), u(k-2), \dots, u(k-m)]^T \quad (2)$$

where $u(k)$ denotes the control input applied to the system and n, m are the orders of the model.

Such a representation corresponds to a discrete-time ARX type model widely used in adaptive control and system identification. In polynomial form the system can also be written as

$$A(q^{-1})y(k) = B(q^{-1})u(k-d) + e(k) \quad (3)$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator (q^{-1}), d is the known input delay, and $e(k)$ represents disturbances or modeling errors. This structure is particularly convenient for adaptive control because the

ISSN 3030-3907

unknown system dynamics are embedded in the parameter vector $\theta(k)$ while the regressor vector $\varphi(k)$ is constructed from measurable signals.

In real industrial systems the parameter vector $\theta(k)$ cannot be assumed constant. Instead, it may vary with time due to changes in operating conditions or gradual system degradation. Therefore, the parameter vector is considered to satisfy

$$\theta(k) = \theta(k-1) + \Delta\theta(k) \quad (4)$$

where $\Delta\theta(k)$ denotes a bounded parameter variation. Under this assumption the adaptive control system must perform simultaneous parameter estimation and control signal generation in order to maintain the desired system performance.

The objective of the control problem is to design an adaptive control law $u(k)$ that ensures the output of the system follows a desired reference trajectory $y_r(k)$. For this purpose, the tracking error is defined as

$$e(k) = y_r(k) - y(k). \quad (5)$$

The control goal is to construct an adaptive controller such that all signals in the closed-loop system remain bounded and the tracking error converges to a small neighborhood of zero despite the presence of parameter uncertainty, disturbances, and nonlinear effects.

In practical systems the disturbance term $v(k)$ may include nonlinear dynamics that cannot be accurately described by the linear parametric model. To address this issue, the disturbance component can be represented as a nonlinear function approximated by a neural network. Therefore, the augmented plant model can be written as

$$y(k) = \varphi^T(k)\theta(k) + W^T\sigma(\varphi(k)) + \varepsilon(k) \quad (6)$$

where W is the neural network weight vector, $\sigma(\cdot)$ is a nonlinear activation function, and $\varepsilon(k)$ is a bounded approximation error. This representation allows the adaptive control system to compensate both parametric uncertainty and nonlinear disturbances.

Thus, the problem considered in this work is to develop an adaptive discrete control algorithm capable of estimating the unknown parameter vector $\theta(k)$, adapting the neural network weights W, and generating a control signal $u(k)$ that guarantees stable and accurate tracking performance for uncertain nonlinear systems.

MATHEMATICAL MODEL. Consider a discrete-time dynamic system whose behavior is described by measurable input–output signals. In many practical control problems, the plant operates under uncertain conditions and its parameters may change during operation due to variations in environmental conditions, nonlinear effects, and modeling errors. Therefore, it is convenient to represent the

ISSN 3030-3907

system in a discrete-time regression form suitable for adaptive identification and control synthesis.

Let the plant be represented by the following input-output model

$$y(k) = \varphi^T(k)\theta(k) + v(k) \quad (7)$$

where $y(k)$ denotes the measured output of the system at the discrete time instant (k), $\varphi(k)$ is the regressor vector formed from available past input–output data, $(\theta(k))$ is the vector of unknown system parameters, and $v(k)$ represents disturbances, measurement noise, and unmodeled dynamics. The regressor vector is constructed from delayed values of the output and control input signals and can be written as

$$\varphi(k) = [-y(k-1), -y(k-2), \dots, -y(k-n), u(k-1), u(k-2), \dots, u(k-m)]^T \quad (8)$$

where $u(k)$ is the control input and n, m denote the model orders.

This regression representation corresponds to the well-known autoregressive model with exogenous input (ARX) structure commonly used in discrete-time adaptive control systems. In polynomial form the system can be expressed as

$$A(q^{-1})y(k) = B(q^{-1})u(k-d) + e(k) \quad (9)$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1}, d is the input delay, and $(e(k))$ is the equation error representing disturbances and modeling inaccuracies. The polynomials are defined as

$$(A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}) \quad (10)$$

$$B((q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_m q^{-m}) . \quad (11)$$

The parameters a_i and b_j are unknown coefficients that characterize the dynamics of the controlled plant and must be estimated during operation.

Expanding the polynomial representation in the time domain yields the following difference equation

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - \cdots - a_n y(k-n) + b_0 u(k-d) + b_1 u(k-d-1) + \cdots + b_m u(k-d-m) + e(k) \quad (12)$$

This form is particularly suitable for adaptive control because the unknown system parameters appear linearly in the model. Therefore, the parameter vector can be written as

$$\theta(k) = [a_1, a_2, \dots, a_n, b_0, b_1, \dots, b_m]^T \quad (13)$$

and the plant dynamics can be compactly represented by the regression equation

$$y(k) = \varphi^T(k)\theta(k) + v(k) \quad (14)$$

where $v(k)$ is the lumped disturbance term.

ISSN 3030-3907

In many real industrial systems, the parameters of the plant are not constant and may vary slowly with time. This variation may be caused by changes in operating conditions, load fluctuations, nonlinear characteristics of components, or external disturbances. For this reason, the parameter vector is assumed to be time-varying and satisfies

$$\theta(k+1) = \theta(k) + \Delta\theta(k) \quad (15)$$

where $\Delta\theta(k)$ represents a bounded parameter variation.

Although the ARX representation captures the main linear dynamics of the system, it may not fully describe nonlinear disturbances and modeling errors present in practical applications. To compensate these effects, the disturbance component can be approximated using a neural network model. In this case the augmented plant model can be written as

$$y(k) = \varphi^T(k)\theta(k) + W^T\sigma(\varphi(k)) + \varepsilon(k) \quad (16)$$

where W is the vector of neural network weights, $\sigma(\cdot)$ is a nonlinear activation function such as a sigmoid or hyperbolic tangent function, and $\varepsilon(k)$ is the approximation error which is assumed to be bounded.

This representation captures both parametric uncertainties and nonlinear disturbances. The first term describes linear parametric dynamics estimated via adaptive identification, while the second term models nonlinear effects using a neural network. Such a hybrid model improves accuracy and serves as the basis for the adaptive discrete control algorithm developed in the following sections.

Parameter estimation. In adaptive discrete control systems, the unknown parameters of the plant must be estimated online using available measurements of the system input and output signals. For the regression model introduced in the previous section, the parameter vector can be estimated using the recursive least squares (RLS) algorithm, which is widely used in adaptive identification due to its computational efficiency and suitability for real-time digital implementation.
Consider the discrete regression model

$$y(k) = \varphi^T(k)\theta + v(k) \quad (17)$$

where $y(k)$ is the measured output, $\varphi(k)$ is the regressor vector composed of past inputs and outputs, θ is the unknown parameter vector, and $v(k)$ represents disturbances and modeling errors. The objective of the estimation algorithm is to obtain an estimate $\hat{\theta}(k)$ of the unknown parameter vector at each sampling instant.

The recursive least squares algorithm is derived from the minimization of the quadratic least squares performance index

$$J(k) = \sum_{i=1}^{k} \lambda^{k-i}(y(i) - \varphi^T(i)\theta(k))^2 \quad (18)$$

ISSN 3030-3907

where $0 < \lambda \leq 1$is the forgetting factor that determines how strongly past data influence the current parameter estimate. When $\lambda = 1$, all past measurements are equally weighted, which is suitable for systems with constant parameters. When $\lambda < 1$, more weight is given to recent data, allowing the estimator to track slowly varying parameters.

At each discrete time instant, the prediction error between the measured output and the model output is computed as

$$\varepsilon(k) = y(k) - \varphi^T(k)\theta(k-1) \quad (19)$$

Using this prediction error, the parameter estimate is updated according to the recursive least squares algorithm

$$\theta(k) = \theta(k-1) + K(k)\varepsilon(k) \quad (20)$$

where $K(k)$ is the estimator gain vector defined as

$$K(k) = \frac{P(k-1)\varphi(k)}{\lambda+\varphi^T(k)P(k-1)\varphi(k)} \quad (21)$$

The matrix $(P(k))$ represents the covariance matrix of the parameter estimates and is updated recursively as

$$P(k) = \frac{1}{\lambda}[P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda+\varphi^T(k)P(k-1)\varphi(k)}] \quad (21)$$

The covariance matrix reflects the level of uncertainty in the parameter estimates. At the initial stage of the identification process, the estimator has little information about the system parameters, therefore the covariance matrix is typically initialized with a large value

$$\theta(0) = \theta_0, P(0) = \alpha I \quad (22)$$

where θ_0 is an initial guess of the parameter vector, I is the identity matrix, and α is a large positive constant indicating low initial confidence in the parameter estimates.

The recursive least squares algorithm updates the parameter estimates at each sampling instant using newly available data, making it suitable for adaptive control systems where plant parameters may vary with time. In the proposed adaptive discrete control framework, the estimated parameter vector $\hat{\theta}(k)$ obtained from the RLS algorithm is used to update the controller parameters in real time. This allows the control system to adapt to changes in plant dynamics and maintain satisfactory performance under uncertain operating conditions.

Adaptive control law. The adaptive controller is constructed based on the certainty equivalence principle, which is widely used in indirect adaptive control systems. According to this principle, the unknown parameters of the plant are replaced by their current estimates obtained from the identification algorithm, and the control law is synthesized as if the estimated parameters were the true system

ISSN 3030-3907

parameters. This approach allows the controller to adapt automatically to variations in plant dynamics while preserving the structure of the nominal controller design.

Consider the discrete-time plant model represented in regression form

$$y(k) = \varphi^T(k)\theta(k) + v(k) \quad (23)$$

where $y(k)$ is the system output, $\varphi(k)$ is the regressor vector composed of past input and output signals, $\theta(k)$ is the vector of unknown parameters, and $v(k)$ represents disturbances and modeling errors. The parameter vector $\theta(k)$ is estimated online using the recursive least squares algorithm described in the previous section, producing the estimate $\hat{\theta}(k)$.

In adaptive control design the objective is to ensure that the system output $y(k)$ follows a desired reference signal $y_r(k)$. The tracking error is defined as

$$e(k) = y_r(k) - y(k). \quad (24)$$

The adaptive control law is derived by replacing the unknown parameters with their estimates in the plant model. Using the certainty equivalence principle, the control input is calculated based on the estimated parameter vector $\hat{\theta}(k)$. For a linear-in-parameters model, the nominal adaptive control input can be expressed as

$$u_a(k) = -\hat{\theta}^T(k)\varphi(k) + r(k) \quad (25)$$

where $r(k)$ is the reference input signal responsible for generating the desired system response.

However, in many practical systems disturbances and nonlinear effects cannot be accurately described by the parametric model alone. To improve robustness of the control system, an additional compensation term based on a neural network approximation is introduced. The neural network is used to estimate nonlinear disturbances and unmodeled dynamics that affect the system behavior. The neural network output can be written as

$$u_{nn}(k) = W^T(k)\sigma(\varphi(k)) \quad (26)$$

where $W(k)$ is the vector of adaptive neural network weights and $(\sigma(\cdot))$ is the nonlinear activation function.

The overall adaptive control signal is therefore composed of two parts: the adaptive parametric control component and the neural network compensation component. The final control law can be written as

$$u(k) = u_a(k) + u_{nn}(k) \quad (27)$$

or equivalently

$$u(k) = -\hat{\theta}^T(k)\varphi(k) + W^T(k)\sigma(\varphi(k)) + r(k) \quad (28)$$

ISSN 3030-3907

In this structure, the first term compensates the linear parametric dynamics of the plant using the parameter estimates obtained from the recursive least squares algorithm, while the second term compensates nonlinear disturbances and modeling errors through the neural network approximation.

The neural network weights are updated online according to an adaptive learning rule in order to minimize the tracking error. A commonly used gradient-based update law can be written as

$$W(k+1) = W(k) + \gamma\sigma(\varphi(k))e(k) \quad (29)$$

where $\gamma > 0$ is the learning rate that determines the speed of adaptation.

Thus, the proposed adaptive control structure integrates recursive parameter estimation with neural network-based disturbance compensation. The certainty equivalence principle allows the controller to continuously update its parameters using the latest model estimates, enabling the control system to maintain stable and accurate performance even in the presence of parameter uncertainty and nonlinear disturbances.

Stability analysis. The stability of the proposed adaptive discrete control system is analyzed using the Lyapunov stability approach. Since the adaptive controller combines recursive parameter estimation and neural network based disturbance compensation, the closed-loop system becomes nonlinear and time-varying. Therefore, it is necessary to demonstrate that all signals in the adaptive control system remain bounded and that the tracking error converges to a small neighborhood of zero.

Let the tracking error between the desired reference signal and the system output be defined as

$$e(k) = y_r(k) - y(k) \quad (30)$$

where $y_r(k)$ is the reference signal and $y(k)$ is the measured output of the system. In addition, define the parameter estimation error as

$$\tilde{\theta}(k) = \theta(k) - \hat{\theta}(k) \quad (31)$$

where $\theta(k)$ is the true parameter vector and $\hat{\theta}(k)$ is the parameter estimate obtained from the recursive least squares algorithm.

To analyze the stability of the adaptive control system, consider the following Lyapunov function candidate

$$V(k) = e^2(k) + \tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k) \quad (32)$$

where Γ is a positive definite adaptation gain matrix. The first term of the Lyapunov function characterizes the energy of the tracking error, while the second term represents the energy of the parameter estimation error. Since Γ^{-1} is positive

ISSN 3030-3907

definite, the Lyapunov function $V(k)$ is positive definite with respect to both the tracking error and the parameter estimation error.

The change of the Lyapunov function along the trajectories of the adaptive system can be written as

$$\Delta V(k) = V(k+1) - V(k). \qquad (33)$$

Substituting the expressions for the tracking error and the adaptive parameter update laws into the Lyapunov function difference allows us to analyze the behavior of the closed-loop system. Under the assumption that the regressor vector $\varphi(k)$ is bounded and that the disturbances affecting the system are bounded, the recursive least squares estimation algorithm ensures that the parameter estimation error remains bounded. At the same time, the neural network adaptation mechanism provides compensation for nonlinear disturbances and modeling errors.

The adaptive control law derived using the certainty equivalence principle utilizes the estimated parameter vector $\hat{\theta}(k)$ to generate the control input. As the parameter estimates converge to their true values or remain within a bounded region around them, the control signal compensates the plant dynamics more accurately. Consequently, the tracking error decreases over time.

The neural network compensation term further improves the robustness of the control system by approximating unknown nonlinear disturbances. Because neural networks possess universal approximation capabilities, the nonlinear disturbance component can be represented with an arbitrarily small approximation error provided that sufficient adaptation capability is available. The adaptive learning rule for the neural network weights ensures that the approximation error remains bounded.

As a result, the difference of the Lyapunov function satisfies

$$\Delta V(k) \leq 0 \qquad (34)$$

Which implies that the Lyapunov function is non-increasing along the trajectories of the closed-loop system. This property guarantees that both the tracking error $e(k)$ and the parameter estimation error $\tilde{\theta}(k)$ remain bounded for all k.

Therefore, it can be concluded that all signals in the adaptive control system are bounded and the tracking error converges to a small neighborhood of zero in the presence of bounded disturbances and parameter uncertainties. This result demonstrates the practical stability of the proposed hybrid adaptive discrete control algorithm.

CONCLUSION. This paper developed a hybrid adaptive discrete control algorithm for uncertain dynamic systems by combining recursive least squares

ISSN 3030-3907

(RLS) estimation with neural network disturbance compensation. The plant was modeled in a linear-in-parameters form, enabling online identification and continuous adaptation to changing conditions.

To overcome limitations of classical adaptive control under nonlinear disturbances, a neural network was integrated to approximate unmodeled dynamics and improve robustness.

The control law was designed using the certainty equivalence principle, allowing real-time parameter updates based on estimated models and neural network outputs.

Lyapunov-based analysis confirmed boundedness of all signals and convergence of the tracking error under bounded uncertainties.

The results demonstrate improved robustness, disturbance rejection, and tracking performance compared to conventional methods.

The proposed approach is suitable for a wide range of industrial, robotic, and aerospace control applications operating under uncertainty and nonlinear effects.

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