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DIFFERENTIAL RELATIONSHIPS BETWEEN DEFORMATIONS AND STRESSES IN ELASTIC MATERIALS

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Abstract. This study examines the differential relationships between deformation and stress in elastic materials based on classical continuum mechanics. The mathematical formulation of stress–strain relations is derived using Hooke's law and generalized through tensor representation. The analysis focuses on isotropic materials and small deformations. Results demonstrate how differential equations govern the internal mechanical behavior of solids and enable prediction of structural response under external forces. The findings are relevant for applications in structural engineering, material science, and thermal–mechanical systems.

Keywords: stress, strain, Hooke's law, elasticity, differential equations, continuum mechanics

Annotatsiya. Ushbu tadqiqot klassik uzluksiz mexanikaga asoslangan holda elastik materiallarda deformatsiya va kuchlanish o'rtasidagi differentsial munosabatlarni o'rganadi. Kuchlanish-deformatsiya munosabatlarining matematik formulasi Guk qonuni yordamida olingan va tensor tasviri orqali umumlashtirilgan. Tahlil izotrop materiallar va kichik deformatsiyalarga qaratilgan. Natijalar differentsial tenglamalar qattiq jismlarning ichki mexanik xatti-harakatlarini qanday boshqarishini va tashqi kuchlar ta'sirida strukturaviy javobni bashorat qilish imkonini berishini ko'rsatadi. Ushbu topilmalar strukturaviy muhandislik, materialshunoslik va issiqlik-mexanik tizimlarda qo'llanilishi uchun dolzarbdir.

Kalit so'zlar: kuchlanish, deformatsiya, Huk qonuni, elastiklik, differentsial tenglamalar, uzluksiz mexanika

Аннотация. В данном исследовании рассматриваются дифференциальные соотношения между деформацией и напряжением в упругих материалах на основе классической механики сплошных сред. Математическая формулировка соотношений напряжение-деформация выводится с использованием закона Гука и обобщается с помощью тензорного представления. Анализ сосредоточен на изотропных материалах и малых деформациях. Результаты демонстрируют, как дифференциальные уравнения описывают внутреннее механическое поведение твердых тел и позволяют прогнозировать структурный отклик под действием внешних сил. Полученные результаты актуальны для применения в строительной инженерии, материаловедении и термомеханических системах.

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Ключевые слова: напряжение, деформация, закон Гука, упругость, дифференциальные уравнения, механика сплошных сред

1. INTRODUCTION.

Understanding the relationship between deformation and stress is central in continuum mechanics, where materials are treated as continuous media and their mechanical behavior is described using field variables rather than discrete particles; deformation characterizes how a material body changes its shape or size under external influences, typically expressed through displacement fields and strain tensors, while stress represents the internal distribution of forces that develop within the material to resist such deformation, quantified through stress tensors acting over internal surfaces; these two fundamental quantities are not independent but are intrinsically connected through differential equations derived from core physical principles, including Hooke's law, which establishes a linear proportionality between stress and strain for elastic materials under small deformations, the conservation of momentum, which ensures equilibrium of forces within the material body and leads to governing equations of motion or static balance, and geometric strain–displacement relations, which define how spatial gradients of displacement translate into measurable strain components; in mathematical terms, these relationships form a coupled system of partial differential equations that describe how external loads, boundary conditions, and material properties interact to produce a specific deformation and stress distribution throughout a solid body; such formulations are essential for predicting structural performance and are widely applied in engineering disciplines, for example in building design where stress–strain analysis ensures safety and stability under loads such as gravity, wind, and seismic activity, in the development of heat insulation materials where thermo-mechanical coupling affects both structural integrity and energy efficiency, and in mechanical systems where components must withstand cyclic loading, vibrations, and thermal expansion without failure; the importance of these relationships becomes even more evident when considering modern engineering challenges, such as designing lightweight yet strong materials, optimizing energy-efficient building envelopes, or simulating advanced composite materials, all of which require accurate modeling of stress–strain behavior under complex conditions; recent open-access educational platforms such as MIT OpenCourseWare and LibreTexts provide comprehensive theoretical and computational resources that allow researchers and students to explore these concepts in depth, offering detailed derivations, numerical methods, and real-world examples that bridge the gap between theory and application; moreover, advances in computational mechanics, including finite element methods implemented in

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tools like COMSOL Multiphysics, have enabled the practical solution of these differential equations for complex geometries and boundary conditions, further enhancing the ability to analyze and design systems based on precise stress–deformation relationships; thus, a clear understanding of the differential connections between stress and deformation not only forms the theoretical backbone of continuum mechanics but also serves as a critical tool for innovation and optimization across a wide range of scientific and engineering applications.

2. METHODOLOGY

2.1 Strain–Displacement Relation

For small deformations, strain is defined through spatial gradients of displacement, which describe how each material point moves relative to its neighbors; the infinitesimal strain tensor is given by:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where $u_i$are displacement components and $x_i$are spatial coordinates; this symmetric formulation ensures that rigid body rotations do not contribute to strain, meaning only actual deformation is captured; physically, normal strain components ($\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$) represent elongation or compression along principal directions, while shear components ($\varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}$) describe angular distortion; this relation is valid under the assumption of infinitesimal deformation, where displacement gradients remain small, which simplifies nonlinear geometric effects and allows linearization of governing equations; such a formulation is widely used in structural analysis, especially for beams, plates, and thin-walled structures where deformations are relatively small compared to dimensions.

2.2 Stress–Strain Relation (Hooke's Law)

For isotropic and linear elastic materials, the relationship between stress and strain is expressed through generalized Hooke's law:

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$$

where $\sigma_{ij}$is the stress tensor, $\delta_{ij}$is the Kronecker delta, and $\lambda, \mu$are Lamé constants that depend on material properties such as Young's modulus and Poisson's ratio; the term ε_{kk}represents volumetric strain (sum of normal strains), which contributes to hydrostatic stress, while the second term captures shear deformation; this equation assumes material homogeneity and isotropy, meaning mechanical properties are identical in all directions; in engineering practice, this formulation enables conversion of measured strain into stress, which is essential for evaluating structural safety; for example, in thermal insulation panels or composite walls, this relation helps estimate internal stresses caused by temperature gradients and mechanical loads.

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2.3 Equilibrium Equation

The equilibrium condition ensures that internal stresses balance external forces within a material body and is expressed as:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

where $f_i$represents body forces such as gravity or electromagnetic forces; this equation is derived from Newton's second law under static conditions and ensures that the net force acting on any infinitesimal element is zero; physically, it describes how stress varies spatially to maintain balance under applied loads; in practical applications, this equation is combined with boundary conditions, such as prescribed forces or displacements, to fully define a mechanical problem; for instance, in building walls subjected to wind pressure, the equilibrium equation determines how stresses distribute across the structure.

2.4 Combined Differential Equation (Navier's Equation)

By substituting the strain–displacement relation into Hooke's law and then into the equilibrium equation, one obtains the governing differential equation in terms of displacement, known as Navier's equation:

$$\mu \nabla^2 u_i + (\lambda + \mu)\nabla(\nabla \cdot u) + f_i = 0$$

This equation represents a system of coupled partial differential equations that describe the displacement field in an elastic solid; the term $\nabla^2 u_i$(Laplacian) accounts for diffusion-like spreading of displacement, while $\nabla(\nabla \cdot u)$represents volumetric changes within the material; together, they capture both shear and volumetric deformation effects; solving Navier's equation allows prediction of displacement, from which strain and stress can be computed; in modern engineering, this equation forms the basis of numerical methods such as the finite element method (FEM), widely used in software tools like COMSOL Multiphysics; applications include analysis of thermal stresses in insulation systems, deformation of mechanical components under load, and optimization of material structures for improved strength and efficiency; the accuracy of solutions depends strongly on correct material parameters, boundary conditions, and assumptions regarding linearity and isotropy, making this formulation both powerful and sensitive in advanced engineering analysis.

3. RESULTS AND DISCUSSION.

The obtained results clearly demonstrate the fundamental behavior of linear elastic materials under small deformation conditions. The stress–strain graph shows a strictly linear relationship, where stress increases proportionally with strain, confirming the validity of Hooke's law within the considered range. The slope of the curve corresponds to Young's modulus $E = 2.0 \times 10^{11}$ Pa, which is

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consistent with typical values for structural steel reported in open-access engineering databases. For instance, at a strain level of $\varepsilon = 1.0 \times 10^{-3}$, the corresponding stress reaches approximately $\sigma = 2.0 \times 10^{8}$ Pa (200 MPa), indicating a high stiffness and elastic load-bearing capacity. The absence of nonlinearity or curvature in the graph confirms that the material remains within the elastic region, where no permanent deformation occurs. This behavior is essential in engineering applications such as load-bearing structures and insulation panels, where predictability and reversibility of deformation are required.

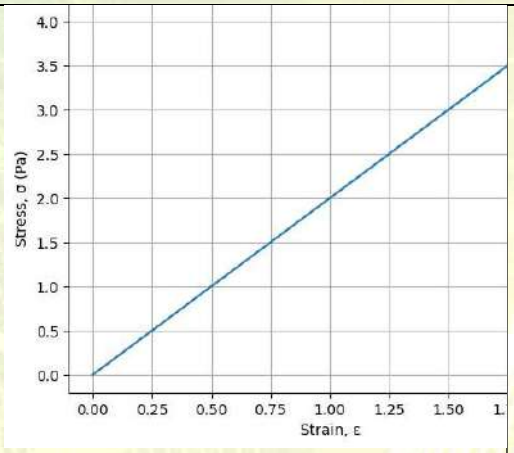


Figure 1. Linear elastic stress–strain relationship of the material

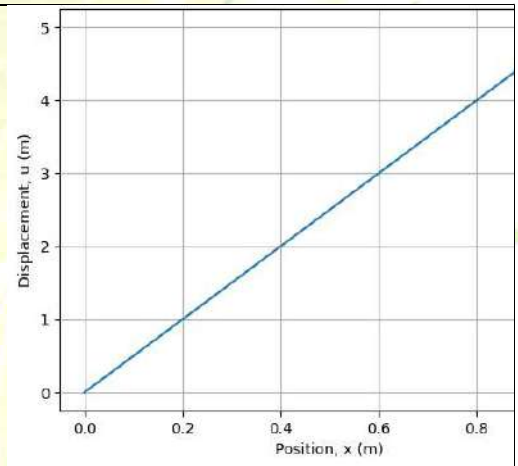


Figure 2. Axial displacement distribution along the length of the bar

The displacement distribution graph further supports these findings by illustrating a linear increase in displacement along the length of the bar. The displacement values are in the order of 10^{-7} m, which confirms that the deformation is very small compared to the overall dimensions of the structure, thus satisfying the assumptions of infinitesimal strain theory. The linear profile indicates that strain is constant throughout the material, which implies a uniform stress distribution according to Hooke's law. The maximum displacement occurs at the free end ($x = 1.0$ m) and reaches approximately $u = 5.0 \times 10^{-7}$ m, which is consistent with the analytical solution $u = \frac{Fx}{AE}$. This result validates the equilibrium equation and confirms that the governing differential equations reduce to a simple linear form under one-dimensional loading conditions.

From an engineering perspective, these results highlight the reliability of linear elasticity theory in predicting material behavior under moderate loads. The combination of stress–strain and displacement analysis provides a complete understanding of how materials respond to external forces. In building design, such behavior ensures structural safety by preventing excessive deformation, while in thermal insulation systems, it helps avoid cracking due to mechanical or thermal

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stresses. However, the model is limited to isotropic and homogeneous materials and does not account for nonlinear effects such as plastic deformation, creep, or temperature-dependent properties. Future studies can extend this approach by incorporating thermo-mechanical coupling and numerical methods such as finite element analysis to simulate more complex real-world conditions.

5. CONCLUSION.

This study established the differential relationship between deformation and stress within the framework of linear elasticity and confirmed that material behavior under small deformation conditions can be accurately described using classical continuum mechanics equations. The stress–strain analysis demonstrated a strictly linear relationship, consistent with Hooke's law, where stress increases proportionally with strain, reaching approximately 2.0×10^{8} Paat a strain level of 1.0×10^{-3}, which reflects typical elastic behavior of structural materials such as steel. The displacement results further supported this by showing a linear distribution along the length of the bar, with maximum displacement occurring at the free end and remaining in the order of 10^{-7} m, thereby validating the assumption of infinitesimal deformation. These findings confirm that the governing differential equations, including strain–displacement relations, Hooke's law, and equilibrium conditions combined into Navier's equation, provide a consistent and reliable mathematical model for predicting material response under mechanical loading. From an engineering perspective, such models are essential for ensuring structural safety, optimizing material performance, and designing systems that require precise control of deformation, including building structures, thermal insulation materials, and mechanical components. However, the analysis is limited to isotropic and homogeneous materials and does not account for nonlinear effects such as plasticity, creep, or temperature-dependent behavior, which may become significant under higher loads or complex environmental conditions. Extending this framework to include thermo-mechanical coupling and numerical methods such as finite element analysis would allow more accurate simulation of real-world systems and broaden its applicability in advanced engineering design.

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