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MODELING HYDROGEOLOGICAL DATA BASED ON THE SPLAYN METHOD

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Abstract. In this study, a cubic Hermit spline model was proposed for the reconstruction of unevenly spaced observation data. The advantage of this model is that it not only restores the level values at specific points but also ensures the continuity of first-order derivatives in transitions between points. This ensures that the hydrogeological level function is physically smooth and hydrodynamically more accurate. The Hermitian spline method has proven to be a high-precision analytical tool for water resource management and in-depth analysis of hydrogeological conditions. Below we present the mathematical expression and algorithm of the Hermit spline model we proposed [2,3].

Keywords: hydrogeological data, bicubic Hermite spline, local interpolation, groundwater level, mathematical modeling, spatial reconstruction, irregular nodes, spline model.

Аннотация. В данном исследовании для восстановления неравномерно расположенных данных наблюдений была предложена кубическая Ermit spline модель. Преимущество данной модели заключается в том, что она не только восстанавливает значения уровней в определенных точках, но и обеспечивает непрерывность производных первого порядка при переходах между точками. Это обеспечивает более гладкую в физическом смысле и гидродинамически более точную картину функции гидрогеологического уровня. Метод эрмитового сплайна показал себя как высокоточный аналитический инструмент в управлении водными ресурсами и глубоком анализе гидрогеологических условий. Ниже приведены математическое выражение и алгоритм предложенной нами модели сплайна Эрмита [2,3].

Ключевые слова: гидрогеологические данные, бикубический сплайн Эрмита, локальная интерполяция, уровень подземных вод, математическое

моделирование, пространственная реконструкция, неравномерные узлы, сплайн-модель.

Annotatsiya. Mazkur tadqiqotda notekis joylashgan kuzatuv ma’lumotlarini tiklash uchun kubik Ermit splayn modeli taklif qilindi. Ushbu modelning afzalligi shundaki, u nafaqat ma’lum nuqtalardagi sath qiymatlarini tiklaydi, balki nuqtalar orasidagi o‘tishlarda birinchi tartibli hosilalarning uzluksizligini ta’minlaydi. Bu esa gidrogeologik sath funksiyasining fizik ma’noda silliq va gidrodinamik jihatdan aniqroq ko‘rinishga ega bo‘lishini ta’minlaydi. Ermit splayn usuli suv resurslarini boshqarishda va gidrogeologik sharoitni chuqur tahlil qilishda yuqori aniqlikdagi analitik vosita ekanligini ko‘rsatdi. Quyida biz taklif etgan Ermit splayn modelining matematik ifodasi va algoritmini keltiramiz [2,3].

Kalit so‘zlar: gidrogeologik ma’lumotlar, bikubik Ermit splayni, lokal interpolatsiya, yer osti suvlari sathi, matematik modellashtirish, fazoviy rekonstruksiya, notekis tugunlar, splayn modeli.

INTRODUCTION.

Hydrogeological data are characterized by continuous changes under the influence of natural and anthropogenic factors. Analyzing seasonal and annual dynamics is of fundamental importance in developing scientifically grounded strategies for the efficient use of water resources. In particular, the change in indicators within one year is the main criterion for determining the water supply and agricultural needs of the region.

When evaluating these processes, it becomes necessary to generalize discrete data obtained from observation wells by spatial points. In practice, such points are located irregularly across the area. Such a distribution of observation points and measurement uncertainties lead to a loss of smoothness and resulting errors in classical interpolation methods. Therefore, to preserve the differential characteristics of the hydrogeological level and flow directions, mathematical models with local and high-order smoothness are used.

METHODOLOGY.

To perform two-dimensional interpolation of the function $f(x,y) \in C^2$ using the Hermitian splay in the rectangular domain $[x_i, x_{i+1}] \times [y_j, y_{j+1}] \subset R^2$, it is necessary to have the values of the function, first-order partial derivatives for x and y, and second-order mixed derivatives at the four nodes of this domain. These nodes and their values are defined as follows:

ISSN 3030-3907

$$\begin{bmatrix} f(x_i,y_j) & f_x(x_i,y_j) & f(x_i,y_{j+1}) & f_x(x_i,y_{j+1}) \\ f_y(x_i,y_j) & f_{xy}(x_i,y_j) & f_y(x_i,y_{j+1}) & f_{xy}(x_i,y_{j+1}) \\ f(x_{i+1},y_j) & f_x(x_{i+1},y_j) & f(x_{i+1},y_{j+1}) & f_x(x_{i+1},y_{j+1}) \\ f_y(x_{i+1},y_j) & f_{xy}(x_{i+1},y_j) & f_y(x_{i+1},y_{j+1}) & f_{xy}(x_{i+1},y_{j+1}) \end{bmatrix}$$

For interpolation, we first normalize the variables:

$$t=\frac{x-x_i}{h_x},\ u=\frac{y-y_j}{h_y},\ h_x=x_{i+1}-x_i,\ h_y=y_{i+1}-y_i,\ t,u\in[0,1]$$

When constructing a bicubic Hermitian spline, we look for the function in the form $P(t,u)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}t^iu^j$, where the coefficients a_{ij} are based on given points, their derivatives, and mixed derivatives.

Calculating the values of the polynomial at points t = 0,1 and u = 0,1, we calculate the coefficients a_{ij}.

Calculating the values of the polynomial at t = 0 and u = 0, we find the coefficients $a_{00},a_{10},a_{01},a_{11}$.

$$P(0,0)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}t^iu^j=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}0^i0^j=a_{00}=f_{00}$$

$$\partial_tP(0,0)=\sum_{i=0}^{3}\sum_{j=0}^{3}ia_{ij}t^{i-1}u^j=\sum_{i=0}^{3}\sum_{j=0}^{3}ia_{ij}0^{i-1}0^j=a_{10}=f_{00}^t$$

$$\partial_uP(0,0)=\sum_{i=0}^{3}\sum_{j=0}^{3}ja_{ij}t^iu^{j-1}=\sum_{i=0}^{3}\sum_{j=0}^{3}ja_{ij}0^i0^{j-1}=a_{01}=f_{00}^u$$

$$\partial_{tu}P(0,0)=\sum_{i=0}^{3}\sum_{j=0}^{3}ija_{ij}t^{i-1}u^{j-1}=\sum_{i=0}^{3}\sum_{j=0}^{3}ija_{ij}0^{i-1}0^{j-1}=a_{11}=f_{00}^{tu}$$

By calculating the values of the polynomial at t = 1, u = 0 and solving the resulting system, we find the coefficients $a_{20},a_{21},a_{30},a_{31}$.

$$P(1,0)=\sum_{i=0}^{3}a_{i0}=a_{00}+a_{10}+a_{20}+a_{30}=f_{10}$$

$$\partial_tP(1,0)=\sum_{i=0}^{3}ia_{i0}=a_{10}+2a_{20}+3a_{30}=f_{10}^t$$

$$\partial_uP(1,0)=\sum_{i=0}^{3}a_{i1}=a_{01}+a_{11}+a_{21}+a_{31}=f_{10}^u$$

$$\partial_{tu}P(1,0)=\sum_{i=0}^{3}ia_{i1}=a_{11}+2a_{21}+3a_{31}=f_{10}^{tu}$$

$$\begin{cases}a_{00}+a_{10}+a_{20}+a_{30}=f_{10}\\ a_{10}+2a_{20}+3a_{30}=f_{10}^t\end{cases}\Rightarrow\begin{cases}a_{30}=f_{10}^t+f_{00}^t+2f_{00}-2f_{10}\\ a_{20}=3f_{10}-3f_{00}-2f_{00}^t-f_{10}^t\end{cases}$$

$$\begin{cases}a_{01}+a_{11}+a_{21}+a_{31}=f_{10}^u\\ a_{11}+2a_{21}+3a_{31}=f_{10}^{tu}\end{cases}\Rightarrow\begin{cases}a_{31}=f_{10}^{tu}+f_{00}^{tu}+2f_{00}^u-2f_{10}^u\\ a_{21}=3f_{10}^u-3f_{00}^u-2f_{00}^{tu}-f_{10}^{tu}\end{cases}$$

ISSN 3030-3907

By calculating the values of the polynomial at t = 0, u = 1 and solving the resulting system, we find the coefficients $a_{02}, a_{03}, a_{12}, a_{13}$.

$$P(0,1) = \sum_{j=0}^{3} a_{0j} = a_{00} + a_{01} + a_{02} + a_{03} = f_{01}$$

$$\partial_t P(0,1) = \sum_{j=0}^{3} a_{1j} = a_{10} + a_{11} + a_{12} + a_{13} = f_{01}^{t}$$

$$\partial_u P(0,1) = \sum_{j=0}^{3} j a_{0j} = a_{01} + 2a_{02} + 3a_{03} = f_{01}^{u}$$

$$\partial_{tu} P(0,1) = \sum_{j=0}^{3} j a_{1j} = a_{11} + 2a_{12} + 3a_{13} = f_{01}^{tu}$$

$$\begin{cases} a_{00} + a_{01} + a_{02} + a_{03} = f_{01} \\ a_{01} + 2a_{02} + 3a_{03} = f_{01}^{u} \end{cases} \Rightarrow \begin{cases} a_{03} = f_{01}^{u} + f_{00}^{u} + 2f_{00} - 2f_{01} \\ a_{02} = 3f_{01} - 3f_{00} - 2f_{00}^{u} - f_{01}^{u} \end{cases}$$

$$\begin{cases} a_{11} + 2a_{12} + 3a_{13} = f_{01}^{tu} \\ a_{10} + a_{11} + a_{12} + a_{13} = f_{01}^{t} \end{cases} \Rightarrow \begin{cases} a_{13} = f_{00}^{tu} + f_{01}^{tu} + 2f_{00}^{t} - 2f_{01}^{t} \\ a_{12} = 3f_{01}^{t} - 3f_{00}^{t} - 2f_{00}^{tu} - f_{01}^{tu} \end{cases}$$

By calculating the values of the polynomial at t = 1, u = 1 and solving the resulting system, we find the coefficients $a_{22}, a_{23}, a_{32}, a_{33}$.

$$P(1,1) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} = f_{11}$$

$$\partial_t P(1,1) = \sum_{i=1}^{3}\sum_{j=0}^{3} i a_{ij} = f_{11}^{t}$$

$$\partial_u P(1,1) = \sum_{i=0}^{3}\sum_{j=1}^{3} j a_{ij} = f_{11}^{u}$$

$$\partial_{tu} P(1,1) = \sum_{i=1}^{3}\sum_{j=1}^{3} ij a_{ij} = f_{11}^{tu}$$

To expand the sum:

$$P(1,1) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} = a_{00} + a_{01} + a_{02} + a_{03} + a_{10} + a_{11} + a_{12} + a_{13} + a_{20} + a_{21} + a_{22} + a_{23} + a_{30} + a_{31} + a_{32} + a_{33} = f_{11}$$

$$\partial_t P(1,1) = \sum_{i=1}^{3}\sum_{j=0}^{3} i a_{ij} = a_{10} + a_{11} + a_{12} + a_{13} + 2a_{20} + 2a_{21} + 2a_{22} + 2a_{23} + 3a_{30} + 3a_{31} + 3a_{32} + 3a_{33} = f_{11}^{t}$$

$$\partial_u P(1,1) = \sum_{i=0}^{3}\sum_{j=1}^{3} j a_{ij} = a_{01} + 2a_{02} + 3a_{03} + a_{11} + 2a_{12} + 3a_{13} + a_{21} + 2a_{22} + 3a_{23} + a_{31} + 2a_{32} + 3a_{33} = f_{11}^{u}$$

$$\partial_{tu} P(1,1) = \sum_{i=1}^{3}\sum_{j=1}^{3} ij a_{ij} = a_{11} + 2a_{12} + 3a_{13} + 2a_{21} + 4a_{22} + 6a_{23} + 3a_{31} + 6a_{32} + 9a_{33} = f_{11}^{tu}$$

Since we calculated the coefficients $a_{00}, a_{01}, a_{02}, a_{03}, a_{10}, a_{11}, a_{12}, a_{13}, a_{20}, a_{21}, a_{30}, a_{31}$ above, we will move these coefficients to the right side of the equality and substitute their values. As a result, we obtain a system of equations depending on $a_{22}, a_{23}, a_{32}, a_{33}$:

ISSN 3030-3907

$$\begin{cases} a_{22}+a_{23}+a_{32}+a_{33}=f_{11}+f_{00}+f_{00}^{t}+f_{00}^{u}-f_{00}^{tu}-f_{01}-f_{01}^{t}-f_{10}-f_{10}^{u} \\ 2a_{22}+2a_{23}+3a_{32}+3a_{33}=f_{11}^{t}+f_{00}^{t}+f_{00}^{tu}-f_{01}^{t}+f_{10}^{t} \\ 2a_{22}+3a_{23}+2a_{32}+3a_{33}=f_{11}^{u}+f_{00}^{u}+f_{00}^{tu}-f_{01}^{u}-f_{01}^{tu}-f_{10}^{u} \\ 4a_{22}+6a_{23}+6a_{32}+9a_{33}=f_{11}^{tu}+f_{00}^{tu}-f_{01}^{tu}-f_{10}^{tu} \end{cases}$$

The solution of the system was as follows:

$$a_{22}=9f_{00}+6f_{00}^{t}+6f_{00}^{u}+4f_{00}^{tu}-9f_{01}-6f_{01}^{t}+3f_{01}^{u}+2f_{01}^{tu}-9f_{10}+3f_{10}^{t}-6f_{10}^{u}+2f_{10}^{tu}+9f_{11}-3f_{11}^{t}-3f_{11}^{u}+f_{11}^{tu}$$

$$a_{23}=-6f_{00}-4f_{00}^{t}-3f_{00}^{u}-2f_{00}^{tu}+6f_{01}+4f_{01}^{t}-3f_{01}^{u}-2f_{01}^{tu}+6f_{10}-2f_{10}^{t}+3f_{10}^{u}-f_{10}^{tu}-6f_{11}+2f_{11}^{t}+3f_{11}^{u}-f_{11}^{tu}$$

$$a_{32}=-6f_{00}-3f_{00}^{t}-4f_{00}^{u}-2f_{00}^{tu}+6f_{01}+3f_{01}^{t}-2f_{01}^{u}-f_{01}^{tu}+6f_{10}-3f_{10}^{t}+4f_{10}^{u}-2f_{10}^{tu}-6f_{11}+3f_{11}^{t}+2f_{11}^{u}-f_{11}^{tu}$$

$$a_{33}=4f_{00}+2f_{00}^{t}+2f_{00}^{u}+f_{00}^{tu}-4f_{01}-2f_{01}^{t}+2f_{01}^{u}+f_{01}^{tu}-4f_{10}+2f_{10}^{t}-2f_{10}^{u}+f_{10}^{tu}+4f_{11}-2f_{11}^{t}-2f_{11}^{u}+f_{11}^{tu}$$

We have already calculated 16 coefficients. Now let's expand the polynomial from the sum.

$$P(t,u)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}t^{i}u^{j}=a_{00}+a_{01}u+a_{02}u^{2}+a_{03}u^{3}+a_{10}t+a_{11}tu+a_{12}tu^{2}+a_{13}tu^{3}+$$

$$+a_{20}t^{2}+a_{21}t^{2}u+a_{22}t^{2}u^{2}+a_{23}t^{2}u^{3}+a_{30}t^{3}+a_{31}t^{3}u+a_{32}t^{3}u^{2}+a_{33}t^{3}u^{3}$$

substituting the coefficients found above, we obtain the following form:

$$\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}t^{i}u^{j}=f_{00}+f_{00}^{u}u+\left(3f_{01}-3f_{00}-2f_{00}^{u}-f_{01}^{u}\right)u^{2}+\left(f_{01}^{u}+f_{00}^{u}+2f_{00}-2f_{01}\right)u^{3}+$$

$$+f_{00}^{t}t+f_{00}^{tu}tu+\left(3f_{01}^{t}-3f_{00}^{t}-2f_{00}^{tu}-f_{01}^{tu}\right)tu^{2}+\left(f_{00}^{tu}+f_{01}^{tu}+2f_{00}^{t}-2f_{01}^{t}\right)tu^{3}+$$

$$+\left(3f_{10}-3f_{00}-2f_{00}^{t}-f_{10}^{t}\right)t^{2}+\left(3f_{10}^{u}-3f_{00}^{u}-2f_{00}^{tu}-f_{10}^{tu}\right)t^{2}u+$$

$$+\left(9f_{00}+6f_{00}^{t}+6f_{00}^{u}+4f_{00}^{tu}-9f_{01}-6f_{01}^{t}+3f_{01}^{u}+2f_{01}^{tu}-9f_{10}+3f_{10}^{t}-6f_{10}^{u}+2f_{10}^{tu}+9f_{11}-3f_{11}^{t}-3f_{11}^{u}+f_{11}^{tu}\right)t^{2}u^{2}+$$

$$+\left(-6f_{00}-4f_{00}^{t}-3f_{00}^{u}-2f_{00}^{tu}+6f_{01}+4f_{01}^{t}-3f_{01}^{u}-2f_{01}^{tu}+6f_{10}-2f_{10}^{t}+3f_{10}^{u}-f_{10}^{tu}-6f_{11}+2f_{11}^{t}+3f_{11}^{u}-f_{11}^{tu}\right)t^{2}u^{3}+$$

$$+\left(f_{10}^{t}+f_{00}^{t}+2f_{00}-2f_{10}\right)t^{3}+\left(f_{10}^{tu}+f_{00}^{tu}+2f_{00}^{u}-2f_{10}^{u}\right)t^{3}u+$$

$$+\left(-6f_{00}-3f_{00}^{t}-4f_{00}^{u}-2f_{00}^{tu}+6f_{01}+3f_{01}^{t}-2f_{01}^{u}-f_{01}^{tu}+6f_{10}-3f_{10}^{t}+4f_{10}^{u}-2f_{10}^{tu}-6f_{11}+3f_{11}^{t}+2f_{11}^{u}-f_{11}^{tu}\right)t^{3}u^{2}+$$

$$+\left(4f_{00}+2f_{00}^{t}+2f_{00}^{u}+f_{00}^{tu}-4f_{01}-2f_{01}^{t}+2f_{01}^{u}+f_{01}^{tu}-4f_{10}+2f_{10}^{t}-2f_{10}^{u}+f_{10}^{tu}+4f_{11}-2f_{11}^{t}-2f_{11}^{u}+f_{11}^{tu}\right)t^{3}u^{3}=$$

group the functional subdivisions corresponding to each of the points $f_{00},f_{00}^{t},f_{00}^{u},f_{00}^{tu},f_{01},f_{01}^{t},f_{01}^{u},f_{01}^{tu},f_{10},f_{10}^{t},f_{10}^{u},f_{10}^{tu},f_{11},f_{11}^{t},f_{11}^{u},f_{11}^{tu}$ and factor each grouped functional subdivision,

$$=f_{00}\left(2t^{3}-3t^{2}+1\right)\left(2u^{3}-3u^{2}+1\right)+f_{00}^{t}\left(t^{3}-2t^{2}+t\right)\left(2u^{3}-3u^{2}+1\right)+$$

$$+f_{00}^{u}\left(2t^{3}-3t^{2}+1\right)\left(u^{3}-2u^{2}+u\right)+f_{00}^{tu}\left(t^{3}-2t^{2}+t\right)\left(u^{3}-2u^{2}+u\right)+$$

$$+f_{01}\left(2t^{3}-3t^{2}+1\right)\left(-2u^{3}+3u^{2}\right)+f_{01}^{t}\left(t^{3}-2t^{2}+t\right)\left(-2u^{3}+3u^{2}\right)+$$

$$+f_{01}^{u}\left(2t^{3}-3t^{2}+1\right)\left(u^{3}-u^{2}\right)+f_{01}^{tu}\left(t^{3}-2t^{2}+t\right)\left(u^{3}-u^{2}\right)+$$

$$+f_{10}\left(-2t^{3}+3t^{2}\right)\left(2u^{3}-3u^{2}+1\right)+f_{10}^{t}\left(t^{3}-t^{2}\right)\left(2u^{3}-3u^{2}+1\right)+$$

$$+f_{10}^{u}\left(-2t^{3}+3t^{2}\right)\left(u^{3}-2u^{2}+u\right)+f_{10}^{tu}\left(t^{3}-t^{2}\right)\left(u^{3}-2u^{2}+u\right)+$$

$$+f_{11}\left(-2t^{3}+3t^{2}\right)\left(-2u^{3}+3u^{2}\right)+f_{11}^{t}\left(t^{3}-t^{2}\right)\left(-2u^{3}+3u^{2}\right)+$$

$$+f_{11}^{u}\left(-2t^{3}+3t^{2}\right)\left(u^{3}-u^{2}\right)+f_{11}^{tu}\left(t^{3}-t^{2}\right)\left(u^{3}-u^{2}\right)$$

ISSN 3030-3907

After factoring, we get basic functions that depend on t and y, which we define as follows:

$$h_{00}(t) = 2t^3 - 3t^2 + 1 \quad h_{00}(u) = 2u^3 - 3u^2 + 1$$
$$h_{10}(t) = t^3 - 2t^2 + t \quad h_{10}(u) = u^3 - 2u^2 + u$$
$$h_{01}(t) = -2t^3 + 3t^2 \quad h_{01}(u) = -2u^3 + 3u^2$$
$$h_{11}(t) = t^3 - t^2 \quad h_{11}(u) = u^3 - u^2$$

After the notation, the bicubic Hermit spline function takes the following form:

$$P(t,u) = f_{00}h_{00}(t)h_{00}(u) + f_{00}^{t}h_{10}(t)h_{00}(u) + f_{00}^{u}h_{00}(t)h_{10}(u) + f_{00}^{tu}h_{10}(t)h_{10}(u) +$$
$$+ f_{01}h_{00}(t)h_{01}(u) + f_{01}^{t}h_{10}(t)h_{01}(u) + f_{01}^{u}h_{00}(t)h_{11}(u) + f_{01}^{tu}h_{10}(t)h_{11}(u) +$$
$$+ f_{10}h_{01}(t)h_{00}(u) + f_{10}^{t}h_{11}(t)h_{00}(u) + f_{10}^{u}h_{01}(t)h_{10}(u) + f_{10}^{tu}h_{11}(t)h_{10}(u) +$$
$$+ f_{11}h_{01}(t)h_{01}(u) + f_{11}^{t}h_{11}(t)h_{01}(u) + f_{11}^{u}h_{01}(t)h_{11}(u) + f_{11}^{tu}h_{11}(t)h_{11}(u)$$

We can write the resulting expression as a sum,

$$S(t,u) = \sum_{i=0}^{1}\sum_{j=0}^{1}\left[f_{ij}h_{0i}(t)h_{0j}(u) + f_{ij}^{t}h_{1i}(t)h_{0j}(u) + f_{ij}^{u}h_{0i}(t)h_{1j}(u) + f_{ij}^{tu}h_{1i}(t)h_{1j}(u)\right]$$

$$S(x,y) = \sum_{p=0}^{1}\sum_{q=0}^{1}\Big[f(x_{i+p}, y_{j+p})h_{0p}(t)h_{0q}(u) + h_x f_x(x_{i+p}, y_{j+p})h_{1p}(t)h_{0q}(u) +$$
$$+ h_y f_y(x_{i+p}, y_{j+p})h_{0p}(t)h_{1q}(u) + h_x h_y f_{xy}(x_{i+p}, y_{j+p})h_{1p}(t)h_{1q}(u)\Big]$$

The surface constructed on this basis has a local and cubic degree, and is defined by a 2D polynomial within each rectangular segment. This polynomial is written as a linear combination of all 16 basic functions. This approach provides geometrically smooth, analytically C^1 or higher class interpolation.

Furthermore, interpolation is constructed while simultaneously preserving surface properties, gradients, and the representation of curved surfaces, allowing it to be applied in geophysical, graphical, hydrogeological, or engineering problems.

This built spline looks as follows:

$$S(t,u) = f_{00}h_{00}(t)h_{00}(u) + f_{00}^{t}h_{10}(t)h_{00}(u) + f_{00}^{u}h_{00}(t)h_{10}(u) + f_{00}^{tu}h_{10}(t)h_{10}(u) +$$
$$+ f_{01}h_{00}(t)h_{01}(u) + f_{01}^{t}h_{10}(t)h_{01}(u) + f_{01}^{u}h_{00}(t)h_{11}(u) + f_{01}^{tu}h_{10}(t)h_{11}(u) +$$
$$+ f_{10}h_{01}(t)h_{00}(u) + f_{10}^{t}h_{11}(t)h_{00}(u) + f_{10}^{u}h_{01}(t)h_{10}(u) + f_{10}^{tu}h_{11}(t)h_{10}(u) +$$
$$+ f_{11}h_{01}(t)h_{01}(u) + f_{11}^{t}h_{11}(t)h_{01}(u) + f_{11}^{u}h_{01}(t)h_{11}(u) + f_{11}^{tu}h_{11}(t)h_{11}(u)$$

where t and y are equal to:

$$t = \frac{x - x_i}{h_x}, \quad u = \frac{y - y_j}{h_y}, \quad h_x = x_{i+1} - x_i, \quad h_y = y_{i+1} - y_i, \quad t,u \in [0,1]$$

$h_{ij}(t)$ and $h_{ij}(u)$ are Hermitian basis functions:

ISSN 3030-3907

$$h_{00}(t) = 2t^3 - 3t^2 + 1 \quad h_{00}(u) = 2u^3 - 3u^2 + 1$$
$$h_{10}(t) = t^3 - 2t^2 + t \quad h_{10}(u) = u^3 - 2u^2 + u$$
$$h_{01}(t) = -2t^3 + 3t^2 \quad h_{01}(u) = -2u^3 + 3u^2$$
$$h_{11}(t) = t^3 - t^2 \quad h_{11}(u) = u^3 - u^2$$

ANALYSIS AND RESULTS.

To evaluate the accuracy and effectiveness of the model, a restoration process is carried out based on the matrix-form values of a specially selected two-dimensional analytical function, and the results are analyzed in comparison with other approaches [4]. At this stage, an algorithm for a two-dimensional Hermit spline model is developed. The values of the selected function are determined in tabular form based on variables x and y with unequal intervals. First of all, this function itself is considered.

$$f(x,y) = \sin(x)\cdot e^{-\cos(y)} + \cos(y)\cdot e^{\sin(x)}$$

where:

x = [-2.0, -1.3, -0.7, 0.0, 1.1, 2.0]
y = [-2.0, -1.4, -0.6, 0.3, 1.2, 2.0]

In Python, we generate the table values of this function and then create a graphical representation based on this table [7].

Returns the two-dimensional grid coordinates based on the coordinates in the x and y vectors. X is a matrix in which each row is a copy of x, and Y is a matrix in which each column is a copy of x. The lattice represented by X and Y coordinates has longitude (y) rows and longitude (x) columns.[8]

Table 1.

Table values of a specially chosen two-variable function

X/Y	-2,0	-1,3	-0,7	0,0	1,1	2,0
-2,0	-1.5462	-1.6196	-1.1952	-0.4161	0.3366	0.3455
-1,4	-0.6987	-0.7481	-0.4543	0.1700	1.1663	1.1891
-0,6	-0.0659	-0.1072	0.1511	0.8253	2.4027	2.4473
0,3	0.0350	-0.0062	0.2538	0.9553	2.6720	2.7215
1,2	-0.4869	-0.5324	-0.2581	0.3624	1.5038	1.5325
2,0	-1.5462	-1.6196	-1.1952	-0.4161	0.3366	0.3455

Let us first present the graphical representation of the function values.

ISSN 3030-3907

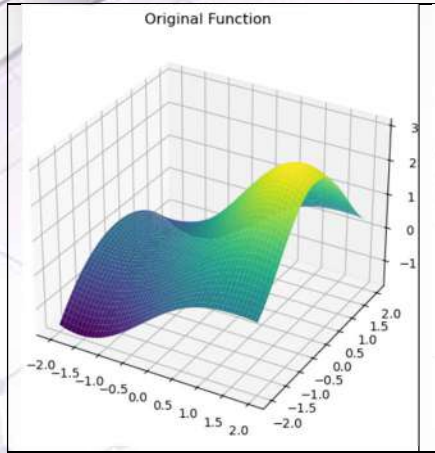


Figure 1.1. Graphical representation of the matrix values of a specially selected two-dimensional function

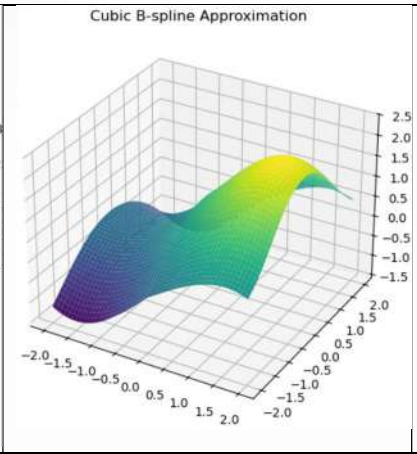


Figure 1.2. Graph of the restoration of matrix values of the function given in Table 1 using a two-dimensional spline model based on a B-spline that does not depend on nodes.

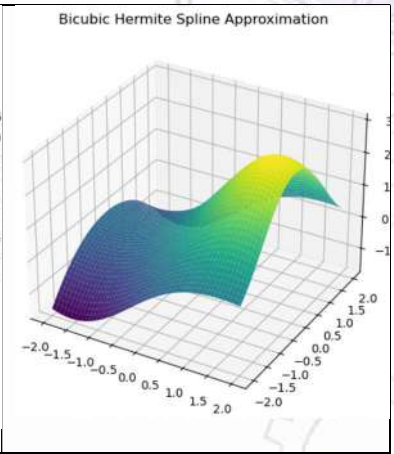


Figure 1.3. Graph of the restoration of the matrix values of the function given in Table 1 using a two-dimensional spline model independent of nodes based on the Hermitian spline.

As can be seen from the images above, the graph of the function restored by the spline model is very close to the original value. We provide a comparative analysis of this in the form of a table in Table 2.

Table 2.

Results of a comparative analysis of the values of the selected function and the values restored by the spline model.

№	x	y	*f(x,y)*	B-spline	Hermite spline	R = |f - BS|	R=|f-HS|
1.	1.5	0.5	2.794294	2.220487	2.777180	0.573807	0.017113
2.	1.6	0.5	2.800109	2.218765	2.782946	0.581345	0.017164
3.	1.7	0.5	2.778037	2.195906	2.763766	0.582130	0.014271
4.	1.8	0.5	2.728852	2.150147	2.719462	0.578704	0.009389
5.	1.9	0.5	2.654253	2.079723	2.649857	0.574529	0.004395
6.	2.0	0.5	2.556741	1.982871	2.554773	0.573870	0.001968
7.	1.5	0.6	2.674872	2.134567	2.657243	0.540305	0.017630
8.	1.6	0.6	2.680440	2.132965	2.662759	0.547475	0.017680
9.	1.7	0.6	2.659309	2.110857	2.644374	0.548452	0.014935
10.	1.8	0.6	2.612214	2.066539	2.601918	0.545674	0.010295
11.	1.9	0.6	2.540760	1.998306	2.535223	0.542454	0.005536
12.	2.0	0.6	2.447312	1.904452	2.444121	0.542860	0.003191

ISSN 3030-3907

13.	1.5	0.7	2.538097	2.037076	2.520683	0.501021	0.017413
14.	1.6	0.7	2.543379	2.035611	2.525917	0.507769	0.017462
15.	1.7	0.7	2.523327	2.014350	2.508435	0.508977	0.014893
16.	1.8	0.7	2.478625	1.971656	2.468079	0.506969	0.010546
17.	1.9	0.7	2.410771	1.905890	2.404692	0.504881	0.006079
18.	2.0	0.7	2.321974	1.815412	2.318118	0.506562	0.003856
19.	1.5	0.8	2.386082	1.929922	2.369822	0.456160	0.016260
20.	1.6	0.8	2.391049	1.928611	2.374744	0.462438	0.016305
21.	1.7	0.8	2.372196	1.908276	0.463920	2.358256	0.013940
22.	1.8	0.8	2.330152	1.867350	0.462802	2.320216	0.009937
23.	1.9	0.8	2.266297	1.804269	0.462028	2.260477	0.005820
24.	2.0	0.8	2.182662	1.717465	0.465196	2.178895	0.003767
					Max error	0.582130	0.017680

Table 3.

Evaluation of absolute and relative errors when restoring the values of a selected function

Method:	Δ_1	Δ_2
B-spline	0,58213	20.95%
Hermite spline	0.01768	0,66%

Where: Δ_1 is the greatest absolute error, and Δ_2 is the greatest relative error. From the table, it can be seen that the Hermitian spline-based model, which does not depend on nodes, has sufficient accuracy when restoring the values of two-dimensional functions on pre-selected unequal intervals. This demonstrates the reliability and accuracy of the proposed model and proves that its application in the field yields good results.

Table 4.

Experimental data from observation wells

x \ y	**0.0**	**5.3**	**10.5**	**15.8**	**21.1**	**26.3**	**31.6**	…	**78.9**	**84.2**	**89.5**	**94.7**	**100**
0.0	40.4	41.4	43.0	43.6	43.8	43.3	42.4	…	39.5	41.9	43.4	43.9	44.4
5.3	39.8	42.1	43.6	43.3	43.4	43.7	42.7	…	40.2	42.2	42.4	43.9	43.5
10.5	40.0	41.4	41.5	42.7	42.7	42.5	41.7	…	39.9	40.8	42.2	42.3	42.9
15.8	40.1	39.9	41.5	41.9	41.0	40.9	40.8	…	40.8	40.3	41.0	41.2	41.7

ISSN 3030-3907

21.1	41.0	39.6	39.5	40.0	39.8	40.3	40.0	…	40.2	39.7	39.7	40.0	39.2
26.3	40.1	39.0	38.5	38.6	38.1	38.3	38.3	…	40.0	39.4	38.5	37.9	38.1
31.6	40.4	38.3	37.6	36.6	37.2	36.6	37.9	…	39.7	38.2	37.8	37.0	36.5
…	…	…	…	…	…	…	…	…	…	…	…	…	…
84.2	40.4	41.9	42.9	43.0	44.2	42.6	42.2	…	40.5	42.1	42.3	43.7	43.8
89.5	39.9	41.6	41.9	42.5	42.5	42.2	41.0	…	40.5	40.8	42.9	42.9	42.4
94.7	39.0	41.4	40.3	40.4	41.5	41.9	41.2	…	39.9	39.8	40.8	40.7	40.6
100.0	40.1	40.3	39.9	38.8	39.0	39.9	39.3	…	40.2	39.7	39.8	39.5	40.0

By reconstructing the experimental data obtained from the observation wells in Table 4 using a local interpolation bicubic spline, it will be possible to analyze groundwater movement and model the level of underground aquifers. By restoring the data in Table 4 through spline interpolation, we will be able to restore the surface shown in Figure 2.

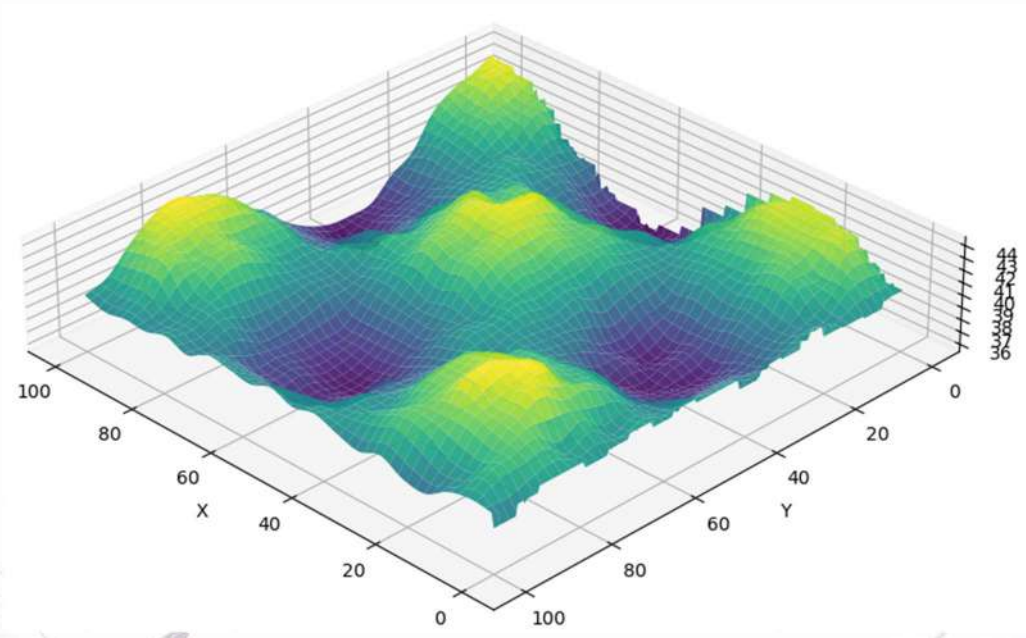


Figure 1. Reconstructed view of the geological level based on a local interpolation bicubic spline.

ISSN 3030-3907

Table 2.

Values of experimental data from observation wells reconstructed by local interpolation bicubic spline.

№	Depth (m)	$S_{3,3}(x,y)$	$\left\|f(x,y)-S_{3,3}(x,y)\right\|$
1	43.715	43.800	0.0799
2	43.699	43.696	0.00358
3	43.687	43.683	0.00749
4	43.677	43.672	0.00807
5	43.668	43.663	0.00716
6	43.658	43.653	0.006596
7	43.645	43.642	0.00822
8	43.629	43.626	0.003862
9	43.606	43.605	0.00537
10	43.576	43.575	0.00458
11	43.537	43.537	0.00332
12	43.488	43.488	0.00217
13	43.434	43.434	0.00434
14	43.376	43.376	0.00352
15	43.313	43.313	0.00349
16	43.245	43.245	0.00465
17	43.169	43.169	0.00078
18	43.086	43.086	0.00354
19	42.996	42.996	0.00437
20	42.896	42.896	0.004004
		Max error:	0.0799

From the table above, we can see that the proposed method approaches the object well and yields good results in the modeling of geological fields.

CONCLUSION.

In this study, a local interpolation bicubic Hermit spline model was developed for high-precision restoration of hydrogeological observation data and spatial level modeling. The main advantage of the proposed approach is that it creates a smooth, physically consistent, and gradient continuous representation of the two-dimensional level, taking into account not only functional values at nodal points but also first-order partial derivatives and mixed derivatives. This allows for a more accurate representation of spatial changes in hydrogeological processes.

ISSN 3030-3907

During the study, the mathematical and computational accuracy of the model was first tested based on a specially selected analytical function. The results obtained showed that the Hermitian spline model yielded significantly higher accuracy compared to the B-spline model when reconstructing two-dimensional functions independent of nodal points. Specifically, the maximum value of the absolute error was 0.01768, which is significantly lower than the value of 0.58213 in the B-spline method. This confirms the high level of local accuracy and smoothness of the proposed model.

At the next stage, the model was applied to data from real hydrogeological observation wells. The results demonstrated the practical effectiveness of this method in restoring the spatial distribution of groundwater levels. The restored surface maintained the general trend of the geological field, local changes, and potential hydrodynamic directions. A small maximum error indicates the model's adaptability and reliability to real objects.

Thus, the developed bicubic Hermit spline model is recommended as an effective mathematical tool for processing non-uniform hydrogeological observation data, modeling groundwater levels, monitoring water resources, and forecasting geological processes. In the future, this approach can be further improved by integrating large volumes of spatial data, dynamic time monitoring systems, and geoinformation platforms.

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