DEVELOPING MATHEMATICAL MODELS FOR THE ELASTOPLASTIC DEFORMATION OF COMPLEX SHAPED AND VOLUMINOUS BODIES.

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Annotatsiya. Uch o'lchovli kartesian koordinata tizimida murakkab shakldagi hajmli jismlarning elastoplastik deformatsiyasining matematik modellari o'rganiladi. Ular, cheklangan farqlar, cheklangan elementlar yoki Vlasov-Kantorovich usullari yordamida hosil qilingan holda, statik va dinamik vaziyatlarda algebraik yoki oddiy differentsial tenglamalarga olib keluvchi tenglamalar tizimlari muhokama qilinadi. 8 tugunli cheklangan elementlarga asoslangan sxemalarning variatsiyalari izoparametrik parallelepiped shaklida o'rganiladi.

Kalit so‘zlar: chekli ayirma, chekli element, Vlasov-Kantorovich usuli, algebraik tenglamalar tizimi.

Аннотация. В трехмерной декартовой системе координат рассматриваются математические модели эластопластической деформации объемных тел со сложными формами. Обсуждаются системы уравнений, которые приводят к алгебраическим или обыкновенным дифференциальным уравнениям в статических и динамических ситуациях, когда они выводятся с использованием конечных разностей, конечных элементов или методов Власова-Канторовича. Исследуются вариации схем на основе 8-узловых конечных элементов в форме изопараметрического параллелепипеда.

Ключевые слова: конечные разности, конечные элементы, метод Власова-Канторовича, система алгебраических уравнений.

Annotation. In a three-dimensional Cartesian coordinate system, the mathematical models of elastoplastic deformation of volumetric bodies with intricate shapes are examined. There is discussion of the systems of equations that result in algebraic or ordinary differential equations in static and dynamic situations when they are derived using finite differences, finite elements, or Vlasov-Kantorovich methods.

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In the form of an isoparametric parallelepiped, variations of schemes based on 8-node finite elements are investigated.

Keywords: finite differences, finite elements, Vlasov-Kantorovich method, system of algebraic equations.

Introduction.

The impact of the relative positions of finite elements on the quality of numerical solutions is not extensively researched, despite the known existence of such effects. By effectively managing computational elements, it is possible to enhance the convergence and accuracy of numerical schemes. The studies referenced in [1] [2] outline the principles for developing and utilizing three-dimensional open work schemes based on the approximation of variational equations, including finite element method schemes. These schemes rely on open meshes of finite elements, which can be created by removing part of a standard isoperimetric parallelepiped. This approach can reduce the number of computational cells by a factor of three or more. Consequently, this significantly cuts down the time needed to resolve issues by minimizing data redundancy in calculating the stress-strain state of the object. Alongside this, it also decreases the time required to establish the system of equations.

LITERATURE REVIEW.

The elastoplastic deformation of materials is a critical area of research, particularly for complex shaped and voluminous bodies encountered in various engineering applications. Understanding the transition from elastic to plastic behavior is essential for predicting material performance under stress.

The foundational theories in elastoplastic modeling include the von Mises yield criterion and the associated flow rule. These theories provide a basis for developing constitutive models that describe the stress-strain relationships in materials. The Ramberg-Osgood equation is frequently used to characterize the nonlinear elastic and plastic behavior, allowing for more accurate predictions in engineering applications.

Finite Element Analysis (FEA) has emerged as a powerful tool for simulating elastoplastic behavior in complex geometries. Research has shown that advanced FEA techniques can effectively capture the intricate deformation patterns and stress distributions in three-dimensional bodies. The incorporation of adaptive meshing and nonlinear solvers further enhances the precision of these simulations.

RESEARCH METHODOLOGY.

Currently, one of the most universal and effective methods widely used for the approximate solution of equations is the finite difference method and the Vlasov-

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Kantorovich variational method. In this case, the Ostrogradsky-Hamilton variational equation can be expressed for each element as follows:

$$\delta\int_{t_1}^{t_2}\left(\int_{v}(\rho[R]dv)\{g\}^{э}-\left(\int_{V}[B]^{'}[D][B]dV\right)\{g\}^{э}-\left(\int_{V}[B]^{'}[\overline{D}]\varpi[B]dV\right)\{g\}^{э}+\int_{S^{э}}[N]^{'}\{P\}dS\right\}dt=0 \quad (1)$$

or by introducing notations

$$[AU]^{э}=\left(\int_{V^{э}}[B]^{'}[D][B]dV\right)$$

$$[AP]^{э}=\left(\int_{V^{э}}[B]^{'}[\overline{D}]\varpi[B]dV\right)$$

$$\{F\}^{э}=\int_{S^{э}}[N]^{'}\{P\}dS$$

here

$[AU]^{e}$ – elastic element stiffness matrix;

$[AP]^{e}$ – additional plastic element to the stiffness matrix;

$\{F\}^{e}$ - node force vector.

here $\sum_{э}\{F_i\}^{э}$ i – the sum of the force components of all elements connected to the node. It is clear that only the elements containing the i-th node contribute to the sum. Now, by combining all the equations, we can express the overall system in the following form:

$([AU]-[AP])\{G\}=\{F\}$, (2)

To solve equations (1)-(2), we will first use the Vlasov-Kantorovich method, which expresses displacements in this form.

$$u_i=u_{i0}(x_1,x_2,x_3,t)+\sum_{j=1}^{N_0}u_{ij}(x_3,t)f_{ij}(x_1,x_2) \quad (3)$$

here

$$\begin{cases} u_{i0}(x_1,x_2,x_3,t)-\text{predefined functions} \\ u_{ij}(x_3,t)-\text{the sought function} \\ f_{ij}(x_1,x_2)-\text{sequence of non}-\text{linear functions} \end{cases}$$

satisfying the boundary conditions (1).

Reduction to a system of ordinary differential and algebraic equations.

Using the relationships between stress and strains, as well as between strains and displacements, the state equations of the given body can be expressed as follows:

$$\rho\ddot{u}_i=\sum_{j=1}^{3}\frac{\partial}{\partial x_j}(\alpha_{jj}H\sigma_{ij})+\sum_{\substack{j=1\\ i\neq j}}^{3}H(\alpha_{ij}\sigma_{ij}-\alpha_{ji}\sigma_{jj})+P_iH \qquad (i=1,2,3) \quad (4)$$

According to the surface boundary conditions of S_p and S_u

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$$(\sigma_{ij} - \sigma_{ij}^{*})\delta u_j\big|_{x_i=const} = 0 \qquad \text{(i,j=1,2,3)} \qquad (5)$$

and with initial conditions

$$u_i\big|_{t=t_0} = \varphi_i \ , \quad \dot{u}_i\big|_{t=t_0} = \psi_i \qquad \text{(i=1,2,3)} \qquad (6)$$

The displacement (3) to (4) and the result

$$\delta T = -\int_v [\rho \sum_{i=1}^{3} \frac{\partial^2 u_i}{\partial t^2} \delta u_i] dV \qquad (7)$$

$$\delta \Pi = \int_v [\sum_{i=1}^{3} \sum_{j\#i}^{3} \sigma_{ij} \delta \varepsilon_{ij}] dV \qquad (8)$$

$$\delta A = \int_v \sum_{i=1}^{3} P_i \delta u_i dV + \int_s \sum_{i=1}^{3} q_i \delta u_i dS \ , \qquad (9)$$

(ρ-material density)

We substitute (7) into (9), then the components of stresses, strains, and displacements are expressed through coordinate functions. The Ostrogradskiy-Hamilton principle (1) and $\delta \int_{t_1}^{t_2} (T - \Pi + A) dt = 0$

Taking into account nnn, we perform integration over the variables x_1 and x_2, and we obtain

$$\int_{t_1}^{t_2} \{ \int_{x_{3,0}}^{x_{3,c}} [\overline{R} Y_{tt} \delta Y + (\overline{A} Y_{x_3} + \overline{B} Y + Q_1) + (\overline{D} Y_{x_3} + \overline{E} Y + Q_2) \delta Y] dx_3 - Q_3 \Big|_{x_3=x_{3,0}}^{x_3=x_{3,c}} = 0 \quad (10)$$

in this case

$$\overline{A}_{ij} = \{\delta_{kl} A_{klij}\}, \overline{R}_{ij} = \{\delta_{kl} R_{klij}\}, \overline{B}_{ij} = \{B_{klij}\}, \overline{D}_{ij} = \{D_{klij}\}, \overline{E}_{ij} = \{E_{klij}\},$$

$$Y = \{Y_i\}, \ Q_k = \{Q_{ki}\}, \ Y_i = \begin{Bmatrix} u_{1,i} \\ u_{2,i} \\ u_{3,i} \end{Bmatrix}, \ Q_{ki} = \begin{Bmatrix} Q_{k,1,i} \\ Q_{k,2,i} \\ Q_{k,3,i} \end{Bmatrix} \ (k,l = 1,2,3; \ i,j = \overline{1, N_0})$$

and $\overline{B}_{1,2,i,j}, \overline{B}_{2,1,i,j}, \overline{D}_{1,2,i,j}, \overline{D}_{2,1,i,j}$ – zero matrices. For the variable X_3, using the rectangular formulas [2], assuming $Y_{k+0.5} = 0.5(Y_{k+1} + Y_k)$, we bring the variational equation into the form (10).

$$\int_{t_1}^{t_2} \sum_{k=0}^{M-1} \left\{ \begin{array}{l} -\overline{R}_{k+0.5} Y_{tt} \delta \dfrac{(Y_{k+1} + Y_k)}{2} + [\overline{A}_{k+0.5} \dfrac{(Y_{k+1} - Y_k)}{h} + \overline{B}_{k+0.5} \dfrac{(Y_{k+1} + Y_k)}{2} + \\ \overline{Q}_{1,k+0.5}] \delta \dfrac{(Y_{k+1} - Y_k)}{h} + [\overline{D}_{k+0.5} \dfrac{(Y_{k+1} - Y_k)}{h} + \overline{E}_{k+0.5} \dfrac{(Y_{k+1} + Y_k)}{2} + \\ \overline{Q}_{2,k+0.5}] \delta \dfrac{(Y_{k+1} + Y_k)}{2} \end{array} \right\} dt + \int_{t_1}^{t_2} Q_3 dt \Big|_{x_3=x_{30}}^{x_3=x_{3c}} = 0 \qquad (11)$$

ISSN 3030-3907

Here, M is the number of intervals in the direction of the variable x_3, and h is the length of the interval step. By setting the δY_k coefficients to zero, we obtain a system of ordinary differential equations (in dynamic problems).

$$-R_k\ddot{Y}_k - B_k Y_{k-1} + C_k Y_k - D_k Y_{k+1} = g_k(t) \qquad (12)$$

with initial information's

$$Y_k(t_0) = \varphi_0, \qquad \dot{Y}_k(t_0) = \psi_0 \quad (k = \overline{0,M}). \qquad (13)$$

We bring the system of second-order differential equations (12) into the following form [3]

$$\dot{Z} = \Phi Z + b \qquad (14)$$

With initial conditions

$$Z\big|_{t=t_0} = Z_0 \qquad (15)$$

in this case

$$Z = \begin{bmatrix} Y \\ \dot{Y} \end{bmatrix}, \qquad \Phi = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}.$$

DISCUSSION AND RESULTS.

In the last decade, the FEM method has become one of the most widely used methods for calculating complex configuration bodies. This is explained by the fact that problems of any complexity can be modeled using this method, that is, systems for solving very high-order equations resulting from arbitrary boundary conditions and configurations of bodies, or related to solving systems of high-order algebraic equations, which can be addressed with the development of computer technologies.

The variational formula of the original problem (4) can be expressed in the following form:

$$\delta\int_{t_1}^{t_2}\left(-\int_v\left[\rho\sum_{i=1}^{3}\frac{\partial^2 u_i}{\partial t^2}\delta u_i\right]dV - \int_v\left[\sum_{i=1}^{3}\sum_{j\#i}^{3}\sigma_{ij}\delta\varepsilon_{ij}\right]dV + \int_v\sum_{i=1}^{3}P_i\delta u_i dV + \int_s\sum_{i=1}^{3}q_i\delta u_i dS\right)dt = 0$$

(16)

In the finite element method, the initial step involves breaking down the given body into finite elements and creating shape functions to estimate the required displacements at the nodal points. Typically, the body is segmented into blocks, which are further divided into finite elements. During this segmentation, it is crucial to take into account all relevant characteristics of the body under study, such as the geometry of block boundaries, material properties, and the locations where applied loads vary. The integration of these blocks is achieved by establishing a straightforward hierarchy of volumes, surfaces, lines, and points. Consequently, a three-dimensional body is depicted as a system of interconnected volumetric elements, with boundary surfaces meeting along boundary lines, which in turn intersect at nodes.

ISSN 3030-3907

The structure may be influenced by changes at the nodes. Therefore, to create the matrix ribbon structure, specific rules must be adhered to when numbering the nodes in the finite element model. If a reduction in the matrix ribbon's width is required, the frontal method can be utilized.

The use of the finite element method is explored, where various problems in complex domains are addressed, making hexagonal finite elements, such as rectangles or oblique prisms, particularly beneficial.

The components of the displacement vector for a specific finite element EEE can be expressed as follows:

$$\{U\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = [IN_1, IN_2, ..., IN_n]\{g\}^э \qquad (17)$$

here

N_i – shape function;

n – Number of nodes in the element;

I – 3x3 dimensional unit matrix;

$\{g\}^э = \{u_{1,1}, u_{2,1}, u_{3,1}, u_{1,2}, u_{2,2}, u_{3,2}, ...u_{1,n}, u_{2,n}, u_{3,n},\}$ - Finite element node displacement vector.

Using the given information, we express the deformation and stress vectors as follows:

$$\{\varepsilon\}^э = [B]\{g\}^э , \qquad (18)$$

$$\{\sigma\}^э = [D]\{\varepsilon\}^э + [\overline{\overline{D}}]\varpi\{\varepsilon\}^э \qquad (19)$$

Here, [B] is the gradient matrix, where $[B]=[B_1,B_2,\ldots.B_n]$,

$$[B_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x_3} \\ \frac{\partial N_i}{\partial x_2} & \frac{\partial N_i}{\partial x_1} & 0 \\ 0 & \frac{\partial N_i}{\partial x_3} & \frac{\partial N_i}{\partial x_2} \\ \frac{\partial N_i}{\partial x_3} & 0 & \frac{\partial N_i}{\partial x_1} \end{bmatrix}, \qquad (20)$$

ISSN 3030-3907

$$[D]=\begin{bmatrix} \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} & \frac{E\mu}{(1+\mu)(1-2\mu)} & \frac{E\mu}{(1+\mu)(1-2\mu)} & 0 & 0 & 0 \\ \frac{E\mu}{(1+\mu)(1-2\mu)} & \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} & \frac{E\mu}{(1+\mu)(1-2\mu)} & 0 & 0 & 0 \\ \frac{E\mu}{(1+\mu)(1-2\mu)} & \frac{E\mu}{(1+\mu)(1-2\mu)} & \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\mu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\mu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\mu)} \end{bmatrix}$$

$$\overline{[D]}=\begin{bmatrix} -\frac{4E}{3(2(1+\mu))} & \frac{2E}{3(2(1+\mu))} & \frac{2E}{3(2(1+\mu))} & 0 & 0 & 0 \\ \frac{2E}{3(2(1+\mu))} & -\frac{4E}{3(2(1+\mu))} & \frac{2E}{3(2(1+\mu))} & 0 & 0 & 0 \\ \frac{2E}{3(2(1+\mu))} & \frac{2E}{3(2(1+\mu))} & -\frac{4E}{3(2(1+\mu))} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{E}{(2(1+\mu))} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{E}{(2(1+\mu))} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{E}{(2(1+\mu))} \end{bmatrix}$$

In this case, the Ostrogadsky-Hamilton variational equation (16) can be written for each element as follows:

$$\delta\int_{t_1}^{t_2}\left(\int_v(\rho[R]dv)\{g\}^э-\left(\int_V[B]^{'}[D][B]dV\right)\{g\}^э-\left(\int_V[B]^{'}[\overline{D}]\varpi[B]dV\right)\{g\}^э+\int_{S^э}[N]^{'}\{P\}dS\right\}dt=0 \quad (21)$$

or by introducing a notation

$$[AU]^э=\left(\int_{V^э}[B]^{'}[D][B]dV\right)$$

$$[AP]^э=\left(\int_{V^э}[B]^{'}[\overline{D}]\varpi[B]dV\right)$$

$$\{F\}^э=\int_{S^э}[N]^{'}\{P\}dS$$

here

[AU]^e – elastic element stiffness matrix;

[AP]^e – addendum to the stiffness matrix for a plastic element;

{F}^{e} - node force vector.

Here, i is the sum of the force components of all elements connected to node i. It is evident that only the elements containing node i contribute to the sum. Now, by combining all the equations from (21), we can express the overall system in the following form:

([AU]-[AP]){G}={F}, (22)

Designated surface loads are automatically accounted for by the external force vector, with displacements being the exception [6].

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Expected Result.

Thus, by determining surface loads and specified displacements, various problems related to the elastic and elastoplastic states of a body with geometric properties can be solved, taking into account different shapes of voids, pits, or additions.

CONCLUSION.

The discussed approach significantly enhances the effectiveness of numerical methods for solving three-dimensional problems in elasticity and plasticity theory. The effectiveness of the proposed schemes is demonstrated by solving test problems and comparing them with solutions based on traditional finite element method schemes.

REFERENCES.

1. Buriyev T., Rasulmakhamadov M.M., *Algorithmic System for Calculating Three-Dimensional Elastic Bodies*, Tashkent: NPO "Cybernetics" of the Academy of Sciences of Uzbekistan, 1994.
2. M. Sikulovich, *Finite Element Method*, Moscow: Stroyizdat, 1993.
3. O. Zenkievich, *Finite Element Method in Engineering*, Moscow: Mir, 1975.
4. A. Samarsky, *Introduction to Numerical Methods*, Moscow: Lan, 2009.
5. Morozov E.M., Nikishkov G.P., *Finite Element Method in Fracture Mechanics*, Moscow: Nauka, 1980.
6. Abdusattorov A., Rasulmakhamadov M.M., "On Solving Spatial Problems of Elasticity Theory by the Finite Difference Method," *Bulletin of TashIIT*, vol. 3, no. 4, pp. 23-27, 2012.
7. V. Vlasov, *Selected Works*, Moscow: Nauka, 1964.
8. N. Kilchevsky, *Course in Theoretical Mechanics: Textbook*, Moscow: Nauka, 1977.

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