ANALITIK FUNKSIYALAR YORDAMIDA ERMIT KUBIK SPLAYN FUNKSIYASINING ANIQLIGINI BAHOLASH

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Anotatsiya. Dunyodagi ko‘plab ilmiy tadqiqotlardan so‘ng aytishimiz mumkinki, raqamli signallar bilan ishlashda splayn funksiyalaridan foydalanish keng qo‘llaniladi va yaxshi natijalarga erishiladi. Hozirgi vaqtda raqamli signallarni tilashning ko‘plab usullari mavjud. Ularga misol qilib Lagranj va Nyuton interpolyatsion usullari, splayn-funksiya, Fure qator va ko‘rsatkichli funksiyalarni yaqinlashtirish usullari, chiziqli tenglamalar tizimini yechishning ortogonal usullari, diskret to‘lqinli transformatsiyalar va bir qator boshqa interpolyatsion usullari kiradi. Tadqiqot davomida Ermit splayn-funksiyasi tanlab olingan. Ushbu maqolada funksiyalarni interpolyatsiya jarayoni lokal kubik splayn-funksiyani berilgan analitik funksiyalarga yaqinlashishi batafsil yoritilgan.

Kalit so‘zlar: bo‘lak polinomial usullar, lokal splayn-funksiyalar, Ermit splayn-funksiya, interpolyatsiya jarayoni, teng oraliqlar uchun kubik splayn, analitik funksiyalar.

Аннотация. После многочисленных научных исследований в мире можно сказать, что при работе с цифровыми сигналами широко используется использование сплайновых функций и достигаются хорошие результаты. В настоящее время существует множество способов пожелать цифровых сигналов. Примеры включают методы интерполяции Лагранжа и Ньютона, сплайновую функцию, методы аппроксимации рядов Фурье и экспоненциальных функций, ортогональные методы решения систем линейных уравнений, дискретные волновые преобразования и ряд других методов интерполяции. В ходе исследования была выбрана отшелушивающая функция Эрмита. В этой статье подробно описывается процесс интерполяции функций в приближении локальной кубической сплайновой функции к заданным аналитическим функциям.

Ключевые слова: методы фрагментарных многочленов, локальные сплайновые функции, Эрмитовая сплайновая функция, процесс

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интерполяции, кубический сплайнинг для равных интервалов, аналитические функции.

Anotation. After many scientific studies around the world, we can say that when working with digital signals, the use of splain functions is widely used and good results are achieved. Currently, there are many ways to wish for digital signals. Examples include LaGrange and Newton interpolation methods, spline-function, Fure series and exponential function approximation methods, orthogonal methods for solving a system of linear equations, discrete wave transformations, and a number of other interpolation methods. During the study, the Hermit splain-function was selected. This article details how the interpolation process of functions approximates the local cubic spline-function to given analytic functions.

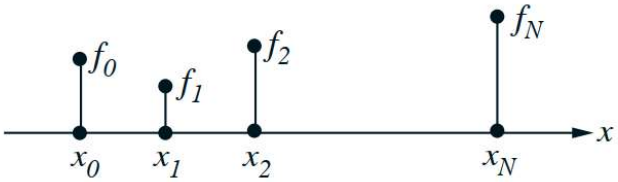
Keywords: slice polynomial methods, local splain-functions, Hermite splain-function, interpolation process, cubic splain for equal intervals, analytic functions.

Kirish

Dunyoda tez rivojlanayotgan axborot-kommunikatsion texnologiyalarida aniqlangan signallarni tiklash, raqamli ishlov berish algoritmlaridan foydalanish va optimal yechimlarni topish uchun yuqori unumli, samarali hisoblash usullarini ishlab chiqish muhim ahamiyat kasb etmoqda. Jumladan lokal splayn-funksiyalarni qurishda hisoblar sonining kamligi, yuqori aniqlik darajasi, raqamli ishlov berish algoritmlarining moslashuvchanligi, optimal differensial va ekstremal xossalari, parametrlarni hisoblashning soddaligi tufayli signallarni raqamli ishlash algoritmlarini yaratishda muhim matematik vosita hisoblanadi [6], [13].

Quyida lokal kubik Ermit splayn usulini qurishni ko‘rib o‘tamiz.

Bizga ma’lumki, $f_i, i = 0, N$ tugun nuqtalari bilan tutashuvchi n- darajali ko‘phad umumiy ko‘rinishini quyidagicha keltirib o‘tishimiz mumkin (1-rasm).



1-rasm. f_i tugun nuqtalari umumiy ko’rinishi

Quriladigan ko‘phadni quyidagi ko‘rinishda qidiramiz:

$$g(x) = \sum_{i=0}^{N} a_i x^i \Rightarrow$$

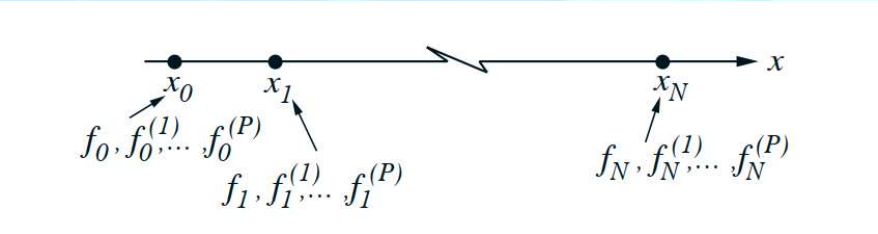
$$g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_N x^N. \qquad (1)$$

Interpolyatsiya jarayonida:

$$g(x_i) = f_i, \quad i = 0,1, \dots, N.$$

N+1 nuqtalarida p-tartibgacha funksiya va uning hosilalariga teng bo‘lgan interpolyatsiya qiluvchi ko‘phadni ishlab chiqamiz.

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Buning uchun biz quyidagilarni belgilab olamiz:

$$g(x_i) = f_i, \quad i = 0,1, \dots, N.$$

$$g^{(1)}(x_i) = f_i^{(1)}, \quad i = 0,1, \dots, N.$$

$$g^{(p)}(x_i) = f_i^{(p)}, \quad i = 0,1, \dots, N. \qquad (2)$$

Bizda (10) va (11) uchun (p+1) va (N+1) umumiy cheklovlar o‘rnatib olamiz.

Yuqoridagi (1) va (2) formulalar yordamida (p+1)(N+1)-1 darajali umumiy ko‘phadni hosil qilamiz (cheklovlar soni interpolyatsiya qiluvchi polinomdagi noma’lumlar koeffitsiyentlar soniga teng bo‘lishi kerak).

$$g(x) = \sum_{i=0}^{(p+1)(N+1)-1} a_i\, x^i. \qquad (3)$$

Ermit interpolyatsion splayn-funksiyasini qurish 3 bosqichda amalga oshiriladi:

1. Interpolyatsiya qiluvchi polinomni aniqlash;
2. Cheklovlarni belgilab olish;
3. Noma’lum koeffitsiyentlarni topish, $a_i, i = 0,\ (p+1)(N+1) - 1$.

[0, 1] oraliq uchun funksiya va uning birinchi hosilasidan o‘tadigan ikkita tugun nuqtadan o‘tuvchi lokal interpolyatsion kubik Ermit splayn-funksiyasini qurib chiqamiz.

	x	f	$f^{(1)}$
x_0	0	f_0	$f_0^{(1)}$
x_1	+1	f_1	$f_1^{(1)}$

bu yerda, p=1 va N+1=2.

1-bosqich. Biz (1+1)(2)=4 chegaraviy shartni belgilab qo‘yamiz. 3-bosqichga muvofiq $(p+1)(N+1)-1$ ni inobatga olgan holda, uchinchi darajali (kubik) interpolyatsiya qiluvchi polinomlarni aniqlaymiz.

$$g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$g^{(1)}(x) = a_1 + 2a_2 x + 3a_3 x^2$$

2-bosqich. Cheklovlarni quyidagi shartlar asosida belgilab olamiz.

$$g(0) = f_0 \quad \Rightarrow \quad a_0 = f_0$$

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$$g(1) = f_1 \quad \Rightarrow \quad a_0 + a_1 + a_2 + a_3 = f_1$$
$$g^{(1)}(0) = f_0^{(1)} \quad \Rightarrow \quad a_1 = f_0^{(1)}$$
$$g^{(1)}(1) = f_1^{(1)} \quad \Rightarrow \quad a_1 + 2a_2 + 3a_3 = f_1^{(1)}$$

3-bosqich. Noma'lum koeffitsiyentlarni matritsa yordamida aniqlaymiz.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_0^{(1)} \\ f_1^{(1)} \end{bmatrix} \quad (4)$$

(4) formuladan foydalanib, noma'lum koefftsiyentlar a_0, a_1, a_2, a_3 ni topamiz.

$a_0 = f_0,$

$a_1 = f_0^{(1)},$

$a_2 = 3f_1 - 3f_0 - f_1^{(1)} - 2f_0^{(1)}$

$a_3 = -2f_1 + 2f_0 + f_0^{(1)} + f_1^{(1)}$

Aniqlangan noma'lum koeffitsiyentlarni (2) formulaga (p+1)(N+1)-1 ni hisobga olib, $g(x)$ quyidagicha tasvirlash mumkin.

$$g(x) = S_3(x)$$
$$= f_0 + f_0^{(1)}x + \left(3f_1 - 3f_0 - f_1^{(1)} - 2f_0^{(1)}\right)x^2$$
$$+ \left(-2f_1 + 2f_0 + f_0^{(1)} + f_1^{(1)}\right)x^3 =$$
$$= f_0 + f_0^{(1)}x + 3f_1x^2 - 3f_0x^2 - f_1^{(1)}x^2 - 2f_0^{(1)}x^2 - 2f_1x^3 + 2f_0x^3 + f_0^{(1)}x^3$$
$$+ f_1^{(1)}x^3 =$$
$$= f_0(2x^3 - 3x^2 + 1) + f_1(-2x^3 + 3x^2) + f_0^{(1)}(x^3 - 2x^2 + x) +$$
$$f_1^{(1)}(x^3 - x^2). \quad (5)$$

$S_3(x)$ ni umumiy shaklda quyidagicha ifodalanishi mumkin:

$$S_3(x) = f_0\alpha_0(t) + f_1\alpha_1(t) + f_0^{(1)}\beta_0(t) + f_1^{(1)}\beta_1(t), \quad (6)$$

bu yerda, $\alpha_i(x), \beta_i(x)$ $i = 0,1$ bazis funksiyalar uchinchi darajali polinomlarni quyidagicha ifodalash mumkin:

$$\alpha_0(t) = 2t^3 - 3t^2 + 1 \qquad \beta_0(t) = t^3 - 2t^2 + t$$
$$\alpha_1(t) = -2t^3 + 3t^2 \qquad \beta_1(t) = t^3 - t^2$$

(6) formuladagi $f_0^{(1)}$ va $f_1^{(1)}$ hisoblash, $f_0^{(1)} = h_i f'_i$, $f_1^{(1)} = h_i f'_{i+1}$ kabi ko'rinishda ifodalash mumkin. $\alpha_i(t), \beta_i(t)$ $i = 0,1$ bazis funksiyalarni soddaroq holatda quyidagicha ifodalash mumkin. Ya'ni, $f_i(t)$, $i = 1,2,3,4$ ko'rinishiga almashtiramiz:

$$\varphi_1(t) = (1-t)^2(1+2t), \qquad \varphi_3(t) = t(1-t)^2,$$

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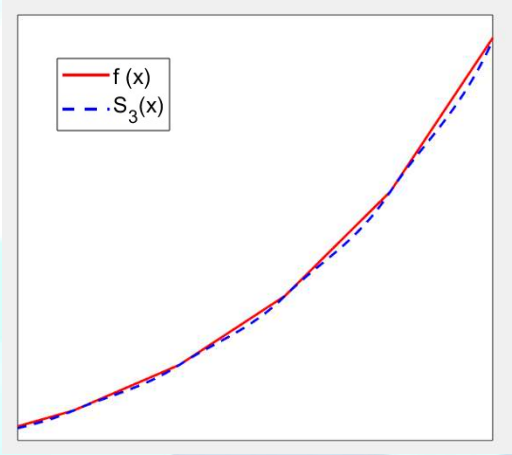
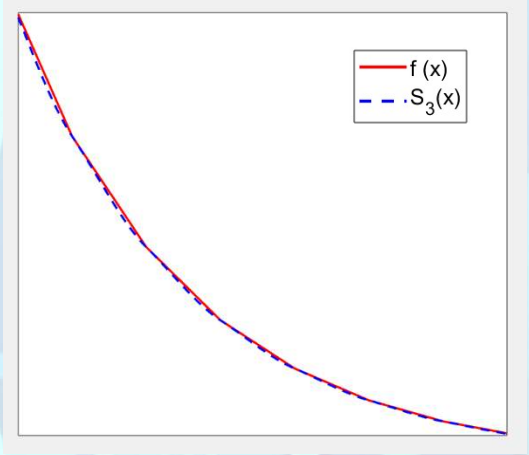
$$\varphi_2(t) = t^2(3-2t), \qquad \varphi_4(t) = -t^2(1-t).$$

Har bir oraliqni hisoblash uchun qadamlarni $h_i = f(x_{i+1}) - f(x_i)$ bilan hisoblanishni hisobga olsak, kubik Ermit splayn-funksiyasi quyidagi ko‘rinishga ega bo‘ladi (7):

$$S_3(x) = \varphi_1(t)f_i(x) + \varphi_2(t)f_{i+1}(x) + \varphi_3(t)h_i f_i'(x) + \varphi_4(t)h_i f_{i+1}'(x) \quad (7)$$

bu yerda, $\varphi_1(t) = (1-t)^2(1+2t)$, $\varphi_3(t) = t(1-t)^2$, $\varphi_2(t) = t^2(3-2t)$, $\varphi_4(t) = -t^2(1-t)$, $h_i = f(x_{i+1}) - f(x_i)$.

Yuqorida qurilishi keltirilgan lokal interpolyatsion kubik Ermit splayn-funksiyaning yaqinlashishi $f(x) = 3^x$ va $f(x) = 3^{-x}$ funksiyalar bilan solishtiramiz:

$f(x) = 3^x$ bo‘lganda	$f(x) = 3^{-x}$ bo‘lganda
	
2-rasm. $f(x) = 3^x$ funksiyani kubik splayn bilan yaqinlashtirish natijalari	**3-rasm. $f(x) = 3^{-x}$ funksiyani kubik splayn bilan yaqinlashtirish natijalari**

Xulosa.

Yuqoridagi rasmlardan ko‘rinadiki, $S_3(x)$ splayn funksiya grafigi yaxshi yaqinlashganini ko‘rish mumkin. Bu esa signallarga raqamli ishlov berishda lokal Ermit splayn matematik modeli, aniqlik ko‘rsatkichi yuqoriligini va qurilish jihatidan soddaligini hisobga olib, biomeditsina, geofizik va boshqa bir o‘lchamli signallarga raqamli ishlashda foydalanish, yuqori samaradorlikka olib kelishi mumkin.

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